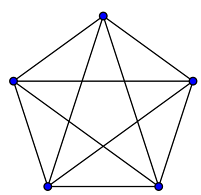
Math 215 - Worksheet 2

September 17, 2018

1. Let x be a real number. Can you find a formula for $\sum_{i=0}^{n} x^i$? Prove by induction that your formula works.

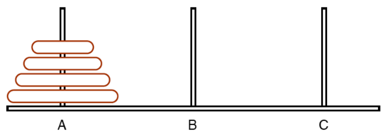
2. Suppose $P_1, P_2, \ldots, P_n$ are n points in the plane with no three collinear. How many lines connect two of these points?

For example, with 5 points:



3. Suppose you have finitely many distinct lines in the plane, $y = m_i x + b_i$ with $1 \leq i \leq n$. These lines partition the plane into finitely many regions. How many colors do you need in order to color in these regions so that no two adjacent regions have the same color? Prove your answer.

4. The *Tower of Hanoi* is a puzzle where the goal is to move 4 washers from Peg A to Peg C. The rules are: Only one washer may be moved from one peg to another at a time; and a washer can never be placed onto a smaller washer.



What is the smallest number of moves in which the puzzle can be solved?

What is this number if the puzzle has n washers instead of 4?

5. What follows is a "proof" of the statement: "There is no horse of a different color."

We prove by induction on the number of horses that any two horses have the same color.

Base case, $n = 1$. There is only one horse; so there are only horses of one color.

Inductive step: $n = k + 1$, assuming inductively that any collection of k horses consists of all horses of the same color.

Given a collection of $k + 1$ horses, we may remove one, and so have a collection of k horses, which by inductive hypothesis, all have the same color. If we show the removed horse has the same color, we are done. So suppose we had removed some other horse. Then we again have by inductive hypothesis a set of k horses, all of which are the same color; this includes the horse we removed originally.

By the principle of induction, any set of n horses consists of all horses of the same color. Since there are finitely many horses, we are done.

What's wrong with this proof?