

# Math 215 - Worksheet 5

October 19, 2018

**1.** Suppose you roll a pair of differently-colored six-sided dice.

- (a) How many total outcomes are there? (Note e.g.  $\blacksquare, \square$  and  $\blacksquare, \square$  are different outcomes)
- (b) How many of these outcomes give a sum of 12?
- (c) A sum of 10?
- (d) What is the most likely sum to roll?

**2.** State abbreviations—such as IL, NJ, or NV—are pairs of letters from the English alphabet, which has 26 letters total.

- (a) How many such pairs are there total?
- (b) How many pairs do not repeat the same letter?
- (c) How many pairs do not contain two vowels?

**3.** For this problem we consider strings of length 5 constructed using only letters from  $\{A, B, C, D, E, F, G, H\}$ . (Example: EGDEA)

- (a) How many such strings are there total?
- (b) How many such strings can be made if repeated letters are not allowed?
- (c) What if the letters have to be in increasing (alphabetical) order? (Example: ABEGH)

4. Suppose  $f : A \rightarrow B$  is a bijection, and that  $A$  is a proper subset of  $B$ . Show  $A$  is infinite.

5. Show there is an injection  $f : A \rightarrow B$  if and only if there is a surjection  $g : B \rightarrow A$ .

6. Let  $X, Y$  be finite sets, and consider the set of injections between them,

$$\text{Inj}(X, Y) = \{f \mid f \text{ is an injection } f : X \rightarrow Y\}.$$

In terms of  $|X|$  and  $|Y|$ , what is  $|\text{Inj}(X, Y)|$ ?