Math 215 - Worksheet 7

November 5, 2018

Recall: A **binary relation on** A is a subset R of $A \times A$. We say A is the **domain of** R, and write $x \, R \, y$ to mean $\langle x, y \rangle \in R$.

- R is **reflexive** if for all $x \in A$, $x \, R \, x$.
- R is **symmetric** if for all $x, y \in A$, if $x \, R \, y$ then $y \, R \, x$.
- R is **transitive** if for all $x, y, z \in A$, if $x \, R \, y$ and $y \, R \, z$, then $x \, R \, z$.

1. Find examples of binary relations R on $\mathbb{Z}$ satisfying each of the following.

(a) R is reflexive and symmetric, but not transitive.

(b) R is reflexive and transitive, but not symmetric.

(c) R is transitive and symmetric, but not reflexive.

An **equivalence relation** on a set A is a binary relation on A that is reflexive, symmetric, and transitive.

2. For each of the following relations R and sets A, determine whether R is an equivalence relation on A. Prove your answers!

(a) $R = \{\langle x, y \rangle \mid x \cdot y > 0\}$, $A = \mathbb{R}$

(b) $x \, R \, y$ iff $5 | (x - y)$, $A = \mathbb{Z}$

(c) $R = \{\langle x, y \rangle \mid |x - y| = 1\}$, $A = \mathbb{Z}$

(d) $x \, R \, y$ iff $y - x \in \mathbb{Q}$, $A = \mathbb{R}$

(e) $\langle a, b \rangle \, R \, \langle c, d \rangle$ iff $a + d = b + c$, $A = \mathbb{N} \times \mathbb{N}$

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function on the reals, and define a relation $R = \{\langle x, y \rangle \mid x = y \vee f(x) = y\}$ on $\mathbb{R}$. For which functions f is this relation R an equivalence relation?