Some suggestions for Assignment 5

Math 300 – Spring 2019 (Hachtman)

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These are some tips for doing the second (harder) problem on the homework.

The analogy you should keep in mind is an oscillatory function, like $\sin x$. Notice that

$$-1 \leq \sin x \leq 1$$

for all x, and sin takes both values 1 and -1 for arbitrarily large values of x.

You should try to understand, using the definition of *limit* in your Calculus text why this tells us that $\lim_{x\to\infty} \sin x$ does not exist. (I suggest looking up the definition and thinking hard about this for 20 minutes. **Just do it!**)

The same sort of reasoning shows that $\lim_{n\to\infty}(-1)^n$ does not exist.

Let's think about the density function ρ now. To simplify notation, for a set A, define for naturals n

$$f_A(n) := \frac{|A \cap [0, n]|}{n + 1}$$

so $0 \leq f_A(n) \leq 1$ for all $n \in \mathbb{N}$, and $\rho(A)$ (if it is defined) is equal to the limit $\lim_{n\to\infty} f_A(n)$. Notice also that the value of $f_A(n)$ just depends on the *initial segment* of A up to n, $A \cap [0, n]$.

Our task is somewhat more difficult than just defining *some* non-convergent sequence. We need to take a step back and define the set A so that $f_A(n)$ is the non-convergent sequence. So try to define A so $f_A(n)$ has something like the oscillatory behavior we saw above for $\sin x$.

$f_A(n)$ can't take both values 0 and 1 infinitely often. But it can take values above, say, 3/4 and below 1/4 infinitely often. (This is what the second hint on the homework prompt is getting at.)

Note the oscillations will take longer and longer to realize the further out we go. I'm thinking of a picture like this: