Applied Linear Algebra
Instructor: Hachtman
Practice Midterm 1

Name: ____________________

UIN: ____________________

You have 50 minutes to complete this exam. No notes, calculators, phones etc. are permitted. **Show all your work.**

1. For each of the following, determine whether the product is defined, and if it is, compute it.

   (a) $\begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} 6 & -2 & 1 \\ -3 & 7 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 4 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -6 & 8 \\ 0 & 14 & 3 & 2 \end{bmatrix}^T$

   (c) $\begin{bmatrix} 1 \\ -6 \\ 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}^T$

   (d) $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^5$

2. Find all values of s for which the system

$$\begin{aligned} 4sx_1 - 10x_2 &= 3 \\ -2x_1 + sx_2 &= 1 \end{aligned}$$

has a unique solution, and give an expression for this solution in terms of s.

3. Find a value of h that makes the collection

$$\left\{ \begin{bmatrix} 1 \\ -2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -3 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ h \\ 9 \end{bmatrix} \right\}$$

linearly dependent. Find a vector that does not belong to the span of the resulting set of vectors.

4. Compute the inverse of the following matrix, or, if it does not exist, say why.

$$\begin{bmatrix} 1 & 4 & -3 & 2 \\ -1 & -3 & 2 & 0 \\ -2 & -7 & 4 & 0 \\ 3 & 12 & -10 & 7 \end{bmatrix}$$

5. Find the matrix of a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ satisfying the following conditions:

   - The vector $\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$ is in the range of T.
   - The vector $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ is *not* in the range of T.

   Explain why no such linear transformation can be one-to-one.

6. Compute the following determinants.

(a) $\begin{vmatrix} 3 & 2 & 2 \\ 1 & 4 & 3 \\ 2 & 5 & 7 \end{vmatrix}$

(b) $\begin{vmatrix} -2 & 6 & 2 & 3 \\ 1 & 3 & 0 & 1 \\ 4 & 2 & 7 & 1 \\ -1 & -2 & 4 & -1 \end{vmatrix}$

(c) $\begin{vmatrix} 2 & 0 & 0 & 0 \\ 2 & -4 & 0 & 0 \\ 17 & 1 & 7 & 0 \\ -23 & 16 & 0 & 1 \end{vmatrix}$

(d) $\det(4I_3)$