Applied Linear Algebra
Instructor: Hachtman
Practice Midterm 2

Name: ______________________

UIN: ______________________

You have 50 minutes to complete this exam. No notes, calculators, phones etc. are permitted. **Show all your work.**

1. Determine whether each of the following sets is a vector subspace. If it is a subspace, show that it is a subspace. If it is not, explain why.

   (a) $W = \left\{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \in \mathbf{R}^4 \;\middle|\; \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is invertible} \right\}.$

   (b) $V = \{ f \in \mathcal{C}(\mathbf{R}) \mid f \text{ is differentiable and } f'(7) = 0 \}.$

2. Let $\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

(a) Define what it means for $\mathcal{B}$ to be a basis for $\mathbf{R}^3$.

(b) Find the change of basis matrix $P_{\mathcal{C} \leftarrow \mathcal{B}}$ from $\mathcal{B}$ to $\mathcal{C}$.

3. Consider the matrix A with row echelon form ref(A):

$$A = \begin{bmatrix} 4 & -6 & 12 & 4 & 10 \\ -2 & 3 & -3 & -3 & -4 \\ 8 & -12 & 18 & 10 & 18 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}, \qquad \text{rref}(A) = \begin{bmatrix} 4 & -6 & 6 & 6 & 8 \\ 0 & 0 & 3 & 0 & 4 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Give a basis for each of the following subspaces associated with A.

(a) Col(A)

(b) Nul(A)

(c) Row(A)

4. The matrix B

$$B = \begin{bmatrix} 2 & 1 & -2 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$$

has only one eigenvalue.

(a) Define what it means for B to be diagonalizable.

(b) Find the eigenvalue λ of B, and give a basis for the eigenspace E_λ.

(c) Show B is not diagonalizable.

5. Let $\langle \mathbf{x}_n \rangle_{n=0}^{\infty}$ be a sequence of vectors in $\mathbf{R}^3$ with $\mathbf{x}_0 = \begin{bmatrix} 0.5 \\ 0.3 \\ 0.2 \end{bmatrix}$ and for all n,

$$\mathbf{x}_{n+1} = \begin{bmatrix} 0.7 & 0.2 & 0.2 \\ 0 & 0.2 & 0.4 \\ 0.3 & 0.6 & 0.4 \end{bmatrix} \mathbf{x}_n.$$

Determine whether the limit $\lim_{n\to\infty} \mathbf{x}_n$ exists, and if it does, compute it.

6. Consider the matrix

$$Q = \begin{bmatrix} -2 & -5 \\ 1 & 4 \end{bmatrix}$$

Find a solution to the differential equation

$$Q\mathbf{x}(t) = \mathbf{x}'(t)$$

satisfying $\mathbf{x}(0) = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$.