Applied Linear Algebra
Instructor: Hachtman
Quiz 1 – 1/12/17

Name: ____________________

UIN: ____________________

This quiz has 2 pages, a front and a back! No notes, calculators, phones etc. are permitted.

1. (2 points) Let $A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 3 & 1 \end{bmatrix}$. Circle the matrices that can be obtained from A using a single elementary row operation.

   (a) $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 3 & \frac{3}{2} \end{bmatrix}$

   (c) $\begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 1 & 3 & 1 \end{bmatrix}$

   Solution: (b) and (d).

   (a) could have only been gotten from multiplying $R1$ by 0, which is not a row operation.

   (b) is obtained by adding $-\frac{1}{2} \cdot R1$ to $R2$.

   (c) is obtained by switching two columns, which is not a row operation.

   (d) is obtained by multiplying $R1$ by $\frac{1}{2}$.

2. (3 points) Find all solutions to the linear system:

$$\begin{aligned} 4x_1 + 2x_3 &= 1 \\ 8x_1 + x_2 + 3x_3 &= 1 \\ 4x_2 - 3x_3 &= -2 \end{aligned}$$

   Solution: The system is represented by the augmented matrix $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 8 & 1 & 3 & 1 \\ 0 & 4 & -3 & -2 \end{bmatrix}$.

   We can put this into RREF as follows:

   Multiply $R1$ by $\frac{1}{4}$: $\begin{bmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{4} \\ 8 & 1 & 3 & 1 \\ 0 & 4 & -3 & -2 \end{bmatrix}$; add $(-8)R1$ to $R2$: $\begin{bmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & -1 & -1 \\ 0 & 4 & -3 & -2 \end{bmatrix}$;

   add $(-4)R2$ to $R3$: $\begin{bmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{4} \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$; add $(-\frac{1}{2})R3$ to $R1$: $\begin{bmatrix} 1 & 0 & 0 & -\frac{3}{4} \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$;

   and add $R3$ to $R2$: $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$.

   We thus have an equivalent matrix corresponding to the linear system $x_1 = -\frac{3}{4}$, $x_2 = 1$, $x_3 = 2$. The unique solution is $(-\frac{3}{4}, 1, 2)$.

3. (2 points) Determine all values of h for which the augmented matrix $\begin{bmatrix} 2 & h & 1 \\ 6 & 2 & -1 \end{bmatrix}$ corresponds to a consistent linear system.

Solution: Adding $(-3)R1$ to $R2$ gives the matrix $\begin{bmatrix} 2 & h & 1 \\ 0 & 2-3h & -4 \end{bmatrix}$. This is in echelon form; and it corresponds to a consistent linear system if and only if there is no row of all zeroes except for a nonzero rightmost entry. So for the system to be consistent we just need $2 - 3h \neq 0$. Therefore the system is consistent as long as $h \neq 2/3$.

4. (3 points) Find the reduced row echelon form of the matrix $\begin{bmatrix} 0 & 1 & 3 & -1 & 1 \\ 1 & 0 & 2 & -1 & -2 \\ 2 & -1 & 1 & 0 & 0 \end{bmatrix}$.

Solution: We row reduce, one step at a time.

Swap $R1$ and $R2$: $\begin{bmatrix} 1 & 0 & 2 & -1 & -2 \\ 0 & 1 & 3 & -1 & 1 \\ 2 & -1 & 1 & 0 & 0 \end{bmatrix}$;

Add $(-2)R1$ to $R3$: $\begin{bmatrix} 1 & 0 & 2 & -1 & -2 \\ 0 & 1 & 3 & -1 & 1 \\ 0 & -1 & -3 & 2 & 4 \end{bmatrix}$;

Add $R2$ to $R3$: $\begin{bmatrix} 1 & 0 & 2 & -1 & -2 \\ 0 & 1 & 3 & -1 & 1 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$;

Note this is in row echelon form but *not* in *reduced* row echelon form since there are nonzero entries above the pivot 1 in the 4th column.

Add $R3$ to $R1$: $\begin{bmatrix} 1 & 0 & 2 & 0 & 3 \\ 0 & 1 & 3 & -1 & 1 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$;

Add $R3$ to $R2$: $\begin{bmatrix} 1 & 0 & 2 & 0 & 3 \\ 0 & 1 & 3 & 0 & 6 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix}$.