Applied Linear Algebra
Instructor: Hachtman
Quiz 13 – 4/28/17

Name: ____________________

UIN: ____________________

This quiz has 2 pages, a front and a back! No notes, calculators, phones etc. are permitted. **Show all your work.**

1. (3 points) Determine whether the matrix $\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ is diagonalizable; explain your answer!

   The matrix is symmetric, and therefore is diagonalizable by a theorem from class. In fact it's diagonalizable by an orthogonal matrix. You could also have found the characteristic polynomial and saw that it had two distinct roots, therefore the dimensions of the eigenspaces add up to 2, like we do when a matrix isn't symmetric, but this is unnecessary.

2. (4 points) Find the singular values of the matrix $\begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}$.

   By definition, these are the square roots of the eigenvalues of the symmetrized matrix

$$A^T A = \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 6 & 13 \end{bmatrix},$$

   which solving the characteristic equation

$$(4 - \lambda)(13 - \lambda) - 36 = \lambda^2 - 17\lambda + 16 = (\lambda - 16)(\lambda - 1)$$

   we have eigenvalues 16, 1; so the singular values are $4, 1$.

3. (3 points) Suppose we want to find the best-fit line for the dataset $(1, 3), (2, 5), (3, 6), (4, 7)$. This line has the form $y = \beta_0 + \beta_1 x$. We should choose β_0, β_1 to be a least-squares solution to which equation? (Circle one.)

(a) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 1 & 5 \\ 1 & 6 \\ 1 & 7 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 5 \\ 6 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \\ 6 \\ 7 \end{bmatrix}$

The answer is (b).

If there were a line of form $y = \beta_0 + \beta_1 x$ running through these four points, it would have to satisfy

$3 = \beta_0 + \beta_1 \cdot 1$

$5 = \beta_0 + \beta_1 \cdot 2$

$6 = \beta_0 + \beta_1 \cdot 3$

$7 = \beta_0 + \beta_1 \cdot 4$

which is precisely what is expressed by the equation in (b).