Applied Linear Algebra
Instructor: Hachtman
Quiz 2 – 1/20/17

Name: ______________________________

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This quiz has 2 pages, a front and a back! No notes, calculators, phones etc. are permitted. **Show all your work.**

1. (4 points) Circle the matrix equations that have a solution.

(a) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 & -1 \\ 6 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 5 & 2 & -1 & 6 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & -9 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 7 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

Solution: (a) and (c) have solutions, since every row of the coefficient matrix contains a pivot column. (b) and (d) do not: For (b), it's because of the last row; for (d), subtract one row from the other to see what's the matter.

2. (2 points) Let $\mathbf{x} = \begin{bmatrix} 5 \\ -1 \\ 6 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix} \in \mathbf{R}^3$. Compute the following vectors:

(a) $\mathbf{x} + \mathbf{y} = \begin{bmatrix} 5 \\ -1 \\ 6 \end{bmatrix} + \begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 5-2 \\ -1+1 \\ 6-2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 4 \end{bmatrix}$

(b) $\frac{1}{3}\mathbf{x} - \frac{2}{3}\mathbf{y} = \frac{1}{3}(\mathbf{x} - 2\mathbf{y}) = \frac{1}{3}\left(\begin{bmatrix} 5 \\ -1 \\ 6 \end{bmatrix} - 2\begin{bmatrix} -2 \\ 1 \\ -2 \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} 5+4 \\ -1-2 \\ 6+4 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ \frac{10}{3} \end{bmatrix}$

3. (1 point) Compute the product:

$$\begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \end{bmatrix} \begin{bmatrix} -13 \\ 0 \\ 26 \end{bmatrix}$$

$$= 13 \left(\begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix} \right) = 13 \begin{bmatrix} -1+0+2 \\ 3+0+4 \end{bmatrix} = 13 \begin{bmatrix} 1 \\ 7 \end{bmatrix} = \begin{bmatrix} 13 \\ 91 \end{bmatrix}$$

4. (3 points) Find all solutions to the matrix equation:

$$\begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Solution: The matrix equation has augmented matrix $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -3 & -1 & 2 & 1 \end{bmatrix}$, so let's put it into RREF.

Add $3 \cdot R1$ to $R2$: $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 5 & 5 & 1 \end{bmatrix}$

Multiple $R2$ by $\frac{1}{5}$: $\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & \frac{1}{5} \end{bmatrix}$

Add $(-2)R2$ to $R1$: $\begin{bmatrix} 1 & 0 & -1 & -\frac{2}{5} \\ 0 & 1 & 1 & \frac{1}{5} \end{bmatrix}$

Now we see that columns 1 and 2 are pivot columns, while column 3 is not, so we know there are infinitely many solutions. x_3 is our free variable. So we have

$$x_1 - x_3 = -\frac{2}{5} \Longrightarrow x_1 = x_3 - \frac{2}{5},$$

and

$$x_2 + x_3 = \frac{1}{5} + x_3 \Longrightarrow x_2 = -x_3 + \frac{1}{5}.$$

So solutions are all vectors in $\mathbb{R}^3$ of the form $\begin{bmatrix} b - \frac{2}{5} \\ -b + \frac{1}{5} \\ b \end{bmatrix}$, where b is any real number.

We can also write the solution set, and the solution set in parametric vector form, as:

$$\left\{ \begin{bmatrix} b - \frac{2}{5} \\ -b + \frac{1}{5} \\ b \end{bmatrix} \mid b \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} -\frac{2}{5} \\ \frac{1}{5} \\ 0 \end{bmatrix} + b \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \mid b \in \mathbb{R} \right\}$$