Applied Linear Algebra
Instructor: Hachtman
Quiz 6 – 2/17/17

Name: ____________________

UIN: ____________________

1. (6 points) Compute the following determinants.

(a) $\begin{vmatrix} 3 & 6 & 2 \\ 2 & 4 & 0 \\ 0 & 1 & 7 \end{vmatrix} = \begin{vmatrix} 3 & 0 & 2 \\ 2 & 0 & 0 \\ 0 & 1 & 7 \end{vmatrix}$,

since column replacements do not change the determinant, and we can expand along the second column: $-0 + 0 - 1(0-4) = 4$.

(b) $\begin{vmatrix} 1 & 4 & -3 & 12 \\ 0 & -2 & 2 & 9 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 5 \end{vmatrix}$

This matrix is upper triangular so the determinant is the product of the diagonal entries: $1(-2)1 \cdot 5 = -10$.

(c) $\begin{vmatrix} 1 & 4 & -3 & 12 \\ 0 & 0 & 1 & 0 \\ 0 & -2 & 2 & 9 \\ 0 & 0 & 0 & 5 \end{vmatrix} = 1 \begin{vmatrix} 0 & 1 & 0 \\ -2 & 2 & 9 \\ 0 & 0 & 5 \end{vmatrix} - 0 + 0 - 0 = 0 - 1 \begin{vmatrix} -2 & 9 \\ 0 & 5 \end{vmatrix} + 0 = 10.$

(d) $\begin{vmatrix} 2 & 7 & 1 & -1 \\ 6 & 12 & 4 & 1 \\ 2 & 7 & 3 & -1 \\ 6 & 13 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 7 & 1 & -1 \\ 6 & 12 & 4 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & -3 & 0 \end{vmatrix}$,

after subtracting the first row from the third and the second from the fourth. Then we can successively clear entries via row replacement:

$$\begin{vmatrix} 2 & 7 & 1 & -1 \\ 6 & 12 & 4 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & -3 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 7 & 0 & -1 \\ 6 & 12 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 & -1 \\ 6 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$
$$= \begin{vmatrix} 2 & 0 & 0 & -1 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} = - \begin{vmatrix} 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{vmatrix}$$

(Note the -1 from the row swap in the last step.) So the determinant is $-2 \cdot 1 \cdot 2 \cdot 4 = -16$.

2. (2 points) Let A be equal to the product of elementary 3×3 matrices,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -9 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

What is $\det(A)$?

Answer: Since $\det(XY) = \det(X)\det(Y)$ for all $n \times n$ matrices X and Y, $\det(A)$ is the product of the determinants of the factors. The second, third, and fifth matrices are row replacement matrices, so have determinant 1. So we have $\det(A) = -1 \cdot 1 \cdot 1 \cdot 4 \cdot 1 = -4$.

3. (2 points) Find the area of the parallelogram with vertices $(1,1), (2,5), (3,4), (4,8)$.

Solution: We translate the parallelogram to have one vertex at the origin, so this is the same as the parallelogram with vertices $\mathbf{0}$, $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

So the area is the absolute value of the determinant of $\begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$, which is $|1 \cdot 3 - 2 \cdot 4| = 5$.