Applied Linear Algebra
Instructor: Hachtman
Quiz 7 – 3/3/17

Name: ______________________________

UIN: ______________________________

This quiz has 2 pages, a front and a back! No notes, calculators, phones etc. are permitted. **Show all your work.**

1. (6 points) For each of the following, determine whether W is a vector subspace of $\mathbf{R}^3$, and if it is not, say why.

   (a) $W = \left\{ \begin{bmatrix} a \\ 1 \\ b \end{bmatrix} \mid a, b \in \mathbf{R} \right\}$

   This is not a subspace, because (for example) it does not contain the zero vector.

   (b) $W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \mid a, b, c \in \mathbf{R} \text{ and } a = b \right\}$

   This is a vector subspace: If $\mathbf{x} = \mathbf{0}$ then $x_1 = x_2 = 0$, so $\mathbf{0} \in W$. If $x, y \in W$ then $x_1 = x_2$ and $y_1 = y_2$; if $\mathbf{z} = \mathbf{x} + \mathbf{y}$ then $z_1 = x_1 + y_1 = x_2 + y_2 = z_2$. So $\mathbf{z} \in W$, and W is closed under addition. Finally, $\mathbf{x} \in W$ and $c \in \mathbf{R}$ implies $cx_1 = cx_2$, so that $c\mathbf{x} \in W$.

   (c) $W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \mid a, b, c \in \mathbf{R} \text{ and } a < c \right\}$

   For any vector $\mathbf{x}$ in W, it's easy to see that $-\mathbf{x} = -1\mathbf{x}$ is not in W. Since W is not closed under scalar multiplication, it is not a vector subspace. (Also $\mathbf{0} \notin W$.)

2. (4 points) Let

$$A = \begin{bmatrix} -1 & -3 & 0 \\ 1 & 1 & 1 \\ 3 & 5 & 2 \end{bmatrix}$$

(a) Find a basis for Col(A).

We row reduce A:

$$\begin{bmatrix} -1 & -3 & 0 \\ 1 & 1 & 1 \\ 3 & 5 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 0 \\ 0 & -2 & 1 \\ 0 & -4 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 3/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

A basis for Col(A) will consist of the columns of A corresponding to the pivot columns of the rref. So a basis is

$$\left\{ \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 5 \end{bmatrix} \right\}$$

(b) Find a basis for Nul(A).

The null space of A is the subspace of solutions to the matrix-vector equation $A\mathbf{x} = \mathbf{0}$. Since we already have the rref of A it's easy to find the solution set: it consists of vectors $\mathbf{x}$ satisfying $x_1 = (-3/2)x_3$ and $x_2 = (1/2)x_3$. So all solutions are multiples of $\begin{bmatrix} -3/2 \\ 1/2 \\ 1 \end{bmatrix}$, and we have basis

$$\mathcal{B} = \left\{ \begin{bmatrix} -3/2 \\ 1/2 \\ 1 \end{bmatrix} \right\}.$$