Applied Linear Algebra
Instructor: Hachtman
Quiz 8 – 3/10/17

Name: ______________________

UIN: ______________________

This quiz has 2 pages, a front and a back! No notes, calculators, phones etc. are permitted. **Show all your work.**

1. (2 points) $\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -6 \end{bmatrix} \right\}$ is a basis for $\mathbf{R}^2$. Find $[\mathbf{e}_1]_{\mathcal{B}}$.

   $[\mathbf{e}_1]_{\mathcal{B}}$ is the unique $\mathbf{c}$ solving the equation

   $$\begin{bmatrix} 2 & 1 \\ 3 & -6 \end{bmatrix} \mathbf{c} = \mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

   We find $\mathbf{c} = \begin{bmatrix} 2/5 \\ 1/5 \end{bmatrix}$.

2. (3 points) Let $W = \{\mathbf{x} \in \mathbf{R}^3 \mid x_3 = 2x_1 + x_2\}$.

   (a) Find a basis for W.

   This space is the null space of the 1×3 matrix $\begin{bmatrix} 2 & 1 & -1 \end{bmatrix}$. Since there is exactly one pivot, we know that $\dim W = 2$. So any linearly independent set of two vectors in W will do, for example: $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

   (b) What is $\dim(W)$?

   2.

3. (2 points) Give an example of an infinite dimensional vector space.

   One could be $\mathbf{P} = \mathbf{R}[t]$, the space of real polynomials.

   Another is $\mathcal{C}(\mathbf{R})$, the space of continuous functions $f : \mathbf{R} \to \mathbf{R}$.

4. (3 points) Let

$$A = \begin{bmatrix} 0 & 3 & 1 & 0 & 3 \\ 1 & 0 & -4 & 3 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

   What is $\dim \mathrm{Nul}(A)$? Justify your answer!

   We can rearrange rows so that the matrix is in row echelon form,

$$A = \begin{bmatrix} 1 & 0 & -4 & 3 & -1 \\ 0 & 3 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

   and now it's clear that there are three pivots. We know by the rank nullity theorem that for an $m \times n$ matrix A,

$$\mathrm{rank}(A) + \dim \mathrm{Nul}(A) = n.$$

   Here the rank is the dimension of the column (equivalently, row) space, which is equal to the number of pivot entries of a row echelon form of A. Since $n = 5$, the dimension of $\mathrm{Nul}(A)$ is $5 - 3 = 2$. (It would also be OK to argue that the dimension is the same as the number of free variables in a general solution to the homogeneous equation, or the number of non-pivot columns.)