Applied Linear Algebra
Instructor: Hachtman
Quiz 9 – 3/17/17

Name: ____________________

UIN: ____________________

This quiz has 2 pages, a front and a back! No notes, calculators, phones etc. are permitted. **Show all your work.**

1. (3 points) Determine the characteristic polynomial of the matrix A:

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 2 & 1 \\ -2 & 0 & 4 \end{bmatrix}$$

Solution: This is $\det(A - \lambda I)$, which we find by expanding along the second column:

$$\begin{vmatrix} 1-\lambda & 0 & -2 \\ 3 & 2-\lambda & 1 \\ -2 & 0 & 4-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 1-\lambda & -2 \\ -2 & 4-\lambda \end{vmatrix} = (2-\lambda)((1-\lambda)(4-\lambda)-4).$$

2. (4 points) The matrix $\begin{bmatrix} 12 & -14 \\ 3 & -1 \end{bmatrix}$ has eigenvalue 6. Find a basis for the corresponding eigenspace.

Solution: This is the same thing as a basis for the null space of $\begin{bmatrix} 12 & -14 \\ 3 & -1 \end{bmatrix} - 6I = \begin{bmatrix} 6 & -14 \\ 3 & -7 \end{bmatrix}$. Row reducing, this is the null space of $\begin{bmatrix} 3 & -7 \\ 0 & 0 \end{bmatrix}$; since the rank of this matrix is 1, we know the dimension of the null space is 1. Any $\mathbf{x}$ satisfying $3x_1 = 7x_2$ will be in the null space. So $\left\{ \begin{bmatrix} 7 \\ 3 \end{bmatrix} \right\}$ is a basis for the eigenspace.

3. (3 points) Let $\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \end{bmatrix} \right\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$. Find the change of basis matrix $P_{\mathcal{C} \leftarrow \mathcal{B}}$.

Solution: We can put the basis vectors in an augmented matrix $[\mathcal{C}|\mathcal{B}]$. We then row reduce to make the left half of the matrix the identity:

$$\left[\begin{array}{cc|cc} -1 & 1 & 4 & -1 \\ 1 & 2 & 2 & 4 \end{array}\right]$$

$$\to \left[\begin{array}{cc|cc} -1 & 1 & 4 & -1 \\ 0 & 3 & 6 & 3 \end{array}\right]$$

$$\to \left[\begin{array}{cc|cc} -1 & 1 & 4 & -1 \\ 0 & 1 & 2 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{cc|cc} -1 & 0 & 2 & -2 \\ 0 & 1 & 2 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{cc|cc} 1 & 0 & -2 & 2 \\ 0 & 1 & 2 & 1 \end{array}\right].$$

Thus the change of basis matrix is $P_{\mathcal{C} \leftarrow \mathcal{B}} = \begin{bmatrix} -2 & 2 \\ 2 & 1 \end{bmatrix}$.