Solutions to True/False HW Questions, Week 3

1.7. 21

(a) True

(b) False, for example consider $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right\}$

(c) True (there are more columns than rows, so too many columns to have a pivot entry in each)

(d) True (why?)

1.8. 21

(a) True

(b) False–the dimension of the domain corresponds to the number of columns, not rows

(c) False

(d) False, at least according to Lay. Some authors use the term "range" to mean what these authors call "codomain." The codomain of a function $f : A \to B$ is B, whereas here the range is defined to be

$$\{f(x) \in B \mid x \in A\},$$

which is equal to B precisely when f is onto. So the statement will fail for any matrix that does not give an onto transformation, e.g. if A is the matrix of all 0's.

(e) It's a theorem that any linear transformation $T : \mathbf{R}^n \to \mathbf{R}^m$ can be obtained as a matrix transformation with $m \times n$ matrix A, and in this restricted setting, the statement is basically true...

...but for more general linear transformations (which we see in Chapter 4) this is not true. This has to do with the *dimension* of these Euclidean spaces being *finite*.

There's also some ambiguity in what the authors mean by "is". Even when a linear transformation can be represented by a matrix, there are many instances where it isn't naturally obtained in this way, and the name "matrix transformation" is sometimes taken to be a description of how the function is presented rather than what it "is". Anyway, I think the answer they want is "False", but in some sense (for transformations of Euclidean space) this is morally "True".

(f) True.

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(a) True

(b) True

(c) False

(d) False

(e) False