MATH 430
HOMEWORK 1

1. Show, for sets A, B, C:
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

2. Suppose $\langle A_i \rangle_{i \in \mathbb{N}}$ is a sequence of sets. Show there is a pairwise disjoint sequence $\langle B_i \rangle_{i \in \mathbb{N}}$ so that $\bigcup A_i = \bigcup B_i$ and $B_i \subseteq A_i$ for all i.

3. Recall a collection $\mathcal{A}$ of subsets of A is a **partition** of A if $\mathcal{A}$ is pairwise disjoint (i.e. $B \cap C = \varnothing$ for all distinct $B, C \in \mathcal{A}$) and $\bigcup \mathcal{A} = A$.
Show $\{\mathcal{A} \mid \mathcal{A} \text{ is a partition of } A\} \sim \{E \subseteq A \times A \mid E \text{ is an equivalence relation on } A\}$.

4. Recall our construction of $(\mathbb{Z}, +, -, \cdot)$ from $(\mathbb{N}, +, \cdot)$; describe a similar construction of the rationals $(\mathbb{Q}, +, -, \cdot)$ from $(\mathbb{Z}, +, -, \cdot)$.

5. Show $A^{B \times C} \sim (A^B)^C$, for all sets A, B, C.

6. Give an explicit bijection between $\mathbb{N}$ and $\mathbb{N} \times \mathbb{N}$; give also an explicit bijection between $\mathbb{N}^{<\mathbb{N}}$ and $\mathbb{N}$.

7. Show if there is a surjection $f : A \to B$, then there is an injection $g : B \to A$. Be sure to make clear where you appeal to the Axiom of Choice.