MATH 430
HOMEWORK 9 — DUE WEDNESDAY, APRIL 6

Enderton: Section 2.6 (p162) 4, 7, 8.

In the following problems, $\mathfrak{N} = (\mathbb{N}; 0, S, <, +, \cdot)$ is the usual structure of the natural numbers in the language of elementary arithmetic.

1. Show that there are, up to isomorphism, uncountably many countable models of $\mathrm{Th}(\mathfrak{N})$. (Hint: Think about prime factorization.)

2. Suppose $\mathfrak{M}_0, \mathfrak{M}_1$ are countable models in the language of arithmetic that satisfy $\mathrm{Th}(\mathfrak{N})$, but neither is isomorphic to $\mathfrak{N}$.

(a) Show $(|\mathfrak{M}_0|; <^{\mathfrak{M}_0})$ and $(|\mathfrak{M}_1|; <^{\mathfrak{M}_1})$ are isomorphic. (Hint: Problem 4 on this homework is relevant.)

(b) Can you describe the order $<^{\mathfrak{M}_0}$?