MATH 430
REVIEW FOR MIDTERM 1

1. (a) State the Cantor-Schroder-Bernstein Theorem.
(b) Show the set $\{f \in \mathbb{N}^{\mathbb{N}} \mid f \text{ is a non-increasing function}\}$ is countable.
(c) Show $\mathbb{R} \sim \mathbb{R}^{\mathbb{N}}$.

2. (a) Define what it means for $(A, \prec)$ to be a linear order.
(b) Is $(\mathcal{P}(\mathbb{N}), \subseteq)$ a linear order?
(c) Define a relation $\preceq$ on $\mathbb{N}^{\mathbb{N}}$ by letting $f \preceq g$ iff $f = g$ or, if n is least such that $f(n) \neq g(n)$, we have $f(n) < g(n)$. Is $(\mathbb{N}^{\mathbb{N}}, \preceq)$ a linear order?

3. (a) State the compactness theorem for propositional logic.
(b) Prove that the compactness theorem is equivalent to the statement that $\Sigma \models \tau$ iff there is some finite subset $\Sigma_0 \subseteq \Sigma$ so that $\Sigma_0 \models \tau$.

4. Recall $\Sigma \vdash \tau$ means there is a deduction of τ from Σ.
(a) Define "deduction of τ from Σ."
(b) Prove the soundness theorem: If $\Sigma \vdash \tau$, then $\Sigma \models \tau$.

5. Let $\sqcup$ denote "exclusive or", that is,

$$A \sqcup B \iff (A \vee B) \wedge \neg(A \wedge B).$$

Is $\{\neg, \sqcup\}$ a complete set of connectives? Prove your answer.

6. Let $G = (X, E)$ be a graph. We say that G is *bipartite* if there is a set $A \subseteq X$ so that vertices in A are only E-adjacent to vertices in $X \setminus A$, and vice versa; that is, xEy implies exactly one of x, y is in A, for all $x, y \in X$.

Let $\mathcal{S}$ be a set of sentence symbols indexed by vertices in G, that is,

$$\mathcal{S} = \{A_x \mid x \in X\}.$$

Give an example of a set Σ of wffs that is satisfiable if and only if G is bipartite.