

MATH 430: FORMAL LOGIC
SPRING 2018
REVIEW PROBLEMS 2

1. Give full definitions of each of the following.
 - tautology
 - \sim
 - deduction
 - \models
 - \mathcal{L} -structure
 - partial order
2. Give full statements of the following named axioms or results.
 - The Axiom of Choice
 - The Deduction Theorem
 - Russel's Paradox
3. Find a disjunctive normal form for $(P \wedge Q) \leftrightarrow (\neg R)$.
4. Let \mathcal{L} be a first order language with binary relation symbol R and constant symbol c . In each of the following \mathcal{L} -formulas, circle all free occurrences of variables.
 - $(\forall x)R(x, c)$
 - $(\forall x \exists y R(x, y)) \wedge R(x, x)$
 - $\exists y (\forall x R(x, y) \rightarrow R(c, x))$
 - $R(c, c)$
 - $R(y, c) \rightarrow \exists y R(x, c)$

Put a check next to each formula that is a sentence.

5. Show $\forall x \forall y (x \leq y \vee y \leq x) \not\models \forall x \forall y \forall z (x \leq y \wedge y \leq z) \rightarrow x \leq z$.
6. Show the following:
 - $\{x \in 2^{\mathbb{N}} \mid \exists i \forall j > i (x(j) = 1)\}$ is countable.
 - $\mathbb{R}^{\mathbb{N}} \sim \mathbb{R}$.
 - Suppose $2 \times A \sim A$. Show A is infinite, and that $\mathbb{N} \times A \sim A$.