MATH 430: FORMAL LOGIC
SPRING 2018
REVIEW PROBLEMS 3

1. Give full definitions of each of the following.

- isomorphism
- axiomatizable class
- $\mathfrak{A} \subseteq \mathfrak{B}$
- finitely satisfiable
- cardinal number
- elementary substructure
- κ-categorical
- wellorder

2. Give full statements of the following named results.

- Compactness Theorem
- Completeness Theorem
- Löwenheim-Skolem Theorem
- Łoś-Vaught Test

3. For each of the following pairs of structures $\mathfrak{A}, \mathfrak{B}$, is $\mathfrak{A}$ a substructure of $\mathfrak{B}$? Is it an elementary substructure? In each case, explain why or why not; if $\mathfrak{A}$ is a substructure of $\mathfrak{B}$ but not an elementary substructure, you must give a formula of the language (possibly with parameters from A) that holds in one but not the other.

(a) $\mathfrak{A} = (M_{2\times 2}(\mathbb{R}); \cdot)$, $\mathfrak{B} = (M_{3\times 3}(\mathbb{R}); \cdot)$
(b) $\mathfrak{A} = (\{A \in M_{2\times 2}(\mathbb{R}) \mid \det(A) \neq 0\}; +)$, $\mathfrak{B} = (M_{2\times 2}(\mathbb{R}); +)$
(c) $\mathfrak{A} = (\{A \in M_{2\times 2}(\mathbb{R}) \mid \det(A) \neq 0\}; \cdot)$, $\mathfrak{B} = (M_{2\times 2}(\mathbb{R}); \cdot)$
(d) $\mathfrak{A} = ((-\infty, 0]; <)$, $\mathfrak{B} = ((-\infty, 1]; <)$
*(e) $\mathfrak{A} = ((-\infty, 0); <)$, $\mathfrak{B} = (\mathbb{R}; <)$

4. Consider the class $\mathcal{E}_{\text{fin}}$ of structures $(A; \approx)$ so that $\approx$ is an equivalence relation on A with finitely many classes. Show $\mathcal{E}_{\text{fin}}$ is not axiomatizable.

5. Show every countable infinite wellorder $\mathfrak{A} = (A; <)$ has an uncountable elementary extension (that is, $\mathfrak{B} = (B; <)$ so that $A \subseteq B$ and $\mathfrak{A} \prec \mathfrak{B}$) that is *not* a wellorder.

6. Recall ω_1 is the least uncountable ordinal.

(a) Show for every countable set $A \subseteq \omega_1$, there is a countable ordinal α so that $A \subseteq \alpha$.
(b) Show there is a countable ordinal α so that $(\alpha; \in) \preceq (\omega_1, \in)$ (elementary substructure).