MATH 430: FORMAL LOGIC
SPRING 2018
REVIEW PROBLEMS 4

1. For each of the following statements, write down a sentence in the language of the given theory that captures the meaning.

(a) (Graphs, $\{E\}$.) Every vertex has degree 2, i.e. shares an edge with exactly two other vertices.
(b) (Graphs, $\{E\}$.)
(c) (Equivalence relations, $\{\approx\}$.) Every equivalence class has exactly 3 elements.
(d) (Rings, $\{0, 1, +, \cdot\}$.) Multiplication is not commutative.
(e) (Linear orders, $\{<\}$.) Every element has an immediate successor.
(f) (Partial orders, $\{<\}$.) The set of predecessors of every element is linearly ordered (by $<$).
(g) (Partial orders, $\{<\}$.) There are no sets of pairwise incomparable elements of size 5.

2. For each sentence you gave in the previous problem, give an example of a structure in which the sentence is satisfied, and one in which it is not.

3. Consider the language $\{0, S\}$ with one constant symbol 0 and unary function symbol S.

Consider the theory T consisting in the following sentences:

- $\forall x \forall y (S(x) = S(y) \to x = y)$
- $\forall x \neg S(x) = 0$
- $\forall x ((\neg x = 0) \to \exists y S(y) = x)$

(a) Describe all countable models of T.
(b) Write down a theory axiomatizing those models of T that contain no "loops", i.e. for no n or x do we have $S^n(x) = x$.
(c) Show, using compactness, that the class of structures from (b) is not finitely axiomatizable.