MATH 430: FORMAL LOGIC
SPRING 2018
HOMEWORK 1

Due Friday, February 2.

1. Write out the truth tables for the following statements:

(a) $P \vee Q \wedge \neg(Q \to P)$
(b) $(P \vee Q) \leftrightarrow \neg(Q \wedge R)$

For each of these, find a statement in disjunctive normal form which is propositionally equivalent to it.

2. Recall a natural number $p \neq 1$ is prime if its only divisors are 1 and itself. Two primes p and q are called *twin primes* if $|p - q| = 2$.

(a) Write down a formula $P(n)$ in the language of arithmetic expressing the property "n is prime."
(b) Write down a sentence expressing the *twin prime conjecture*: "there are infinitely many pairs of twin primes." (You're not meant to write out your formula from part (a), but instead to write $P(n)$ as an abbreviation for it.)

3. Define a binary relation $\sqsubset$ on the natural numbers $\mathbb{N}$ by setting $m \sqsubset n$ iff m is even and n is odd, or m and n are both odd and $m < n$, or m and n are both even and $m < n$.

Find a sentence in the language $\mathcal{L}$ with one binary relation symbol that is true in the structure $(\mathbb{N}, \sqsubset)$, but false in the structure $(\mathbb{N}, <)$. (Hint: Start by trying to draw a picture of the order $\sqsubset$. You should explain why your answer works!)

4. (Extra Credit.) You find yourself in a distant land on your way to deliver a message to the castle. In this land there are three sorts of people: *workers* (who always tell the truth), *businessmen* (who always lie), and *students* (who sometimes lie, and sometimes tell the truth). You come to a fork in the road, near which are standing a worker, a student, and a businessman—but you can't tell which is which. By asking two yes-or-no questions (which you can put to any two of the three individuals), find which path leads to the castle.