

MATH 430: FORMAL LOGIC
SPRING 2018
HOMEWORK 2

Due Friday, February 9.

1. Let T be a first order theory and φ be a formula.
 - (a) Prove that $T \vdash \neg\varphi$ if and only if $T \cup \{\varphi\}$ is inconsistent.
 - (b) Prove that if T is inconsistent, then $T \vdash \psi$ for *all* formulas ψ .
2. Write down a deduction witnessing $\emptyset \vdash \forall x\varphi \rightarrow \exists x\varphi$.
3. Show that no one of the following sentences is logically implied by the other two.
 - (a) $\forall x\forall y\forall z(P(x, y) \rightarrow (P(y, z) \rightarrow P(x, z)))$
 - (b) $\forall x\forall y(P(x, y) \rightarrow (P(y, x) \rightarrow x = y))$
 - (c) $\forall x\exists yP(x, y) \rightarrow \exists y\forall xP(x, y)$
4. Prove $T \models \varphi \wedge \neg\varphi$ if and only if T is unsatisfiable.
5. **(Extra Credit.)** For your little nephew's fourth birthday, you decide to gift a set of number magnets, consisting of one each of the numbers 0 through 9, and the symbols $+$, $-$, $*$, $/$, $=$.
 - (a) Your nephew asks how many equations you can make with these magnets (where “ $-$ ” is allowed to be either binary or unary, and allowing redundancies like e.g. “ $0 = 6 - 3 * 2$ ” and “ $6 - 3 * 2 = 0$ ”). What do you tell him?
 - (b) Suppose your nephew asks how many *true* equations you can make with these magnets. Now what do you do?