MATH 430: FORMAL LOGIC
SPRING 2018
HOMEWORK 6

Due Friday, April 6.

1. A graph $G = (V, E)$ is called *bipartite* if there is a set $B \subseteq V$ so that if x, y are either both in B or both not in B, then $\langle x, y \rangle \notin E$.

A *cycle of length* n in G is a sequence $\langle x_0, x_1, \dots x_n \rangle$ so that $x_0 = x_n$, $x_i \neq x_j$ for $0 < i < j < n$, and $\langle x_i, x_{i+1} \rangle \in E$ for all $i < n$.

(a) Draw two graphs of size 6, one bipartite and one not.

(b) Show a graph is bipartite if and only if there are no cycles of *odd* length in G. (Hint: for the hard direction, define B inductively, starting with a point in each connected component of the graph and moving outwards.)

(c) Show the class of bipartite graphs is axiomatizable.

(d) Show the class of bipartite graphs is *not* finitely axiomatizable.

2. Let E be the set of even integers. Show the structures $(\mathbb{Z}, E)$ and $(\mathbb{R}, E)$, in the language with one unary relation symbol, are elementarily equivalent: $(\mathbb{Z}, E) \equiv (\mathbb{R}, E)$.

3 (Extra credit)**.** Let $P = \{p_1, p_2, p_3, \dots\} = \{2, 3, 5, 7, 11, \dots\}$ be an enumeration of the primes. For naturals k, we let $S^k(0)$ be the term in the language of arithmetic that denotes the k-th successor of 0.

(a) Show, for each $D \subseteq \mathbb{N}$, that there is a nonstandard model of arithmetic $\mathcal{A}$ with an element c so that for all naturals n, $n \in D$ if and only if $\mathcal{A} \models \exists x\, x \cdot S^{p_n}(0) = c$.

(b) Use part (a) to show there are uncountably many models of the theory of true arithmetic up to isomorphism.