MATH 430: FORMAL LOGIC
SPRING 2018
HOMEWORK 7

Due Friday, April 27.

1. Give a derivation witnessing that the function $m(x, y) = x \dot{-} y$ is primitive recursive.

2. Show that the relation $x < y$ is primitive recursive.

3. Give a derivation of the function $\exp(x, y) = x^y$ as a primitive recursive function, taking for granted that $f_0(x, y) = x \cdot y$ is primitive recursive.

4. Show that if a set $R \subseteq \mathbb{N}$ is finite, then it is primitive recursive.

5. Recall p_k is the k-th prime. Show the function $f : \mathbb{N}^{<\mathbb{N}} \to \mathbb{N}$ defined by

$$f(\varnothing) = 0$$
$$f(\langle s_1, \dots, s_k \rangle) = 2^{s_1} \cdot 3^{s_2} \cdot 5^{s_3} \cdot 7^{s_4} \cdot 11^{s_5} \cdots p_k^{s_k+1} - 1$$

is a bijection.

6. As above, p_k denotes the k-th prime.

(a) Show every interval of the form $(n, n! + 1]$ contains a prime number.

(b) Show the function $k \mapsto p_k$ is primitive recursive. (Recall from class that if $R(x)$ is primitive recursive, then so is $f(m, x)$ = "the least $x \le m$ such that $R(x)$, if there is such, and 0 otherwise").