2. Let $U$ be an open subset of $\mathbb{C}$, and let $(f_t)_{t \in T}$ be a locally uniformly absolute summable family of meromorphic functions on $U$. Suppose that the family of derivatives $(f'_t)_{t \in T}$ is also locally uniformly absolute summable. Let $S(z)$ denote the meromorphic function defined on $U$ by

$$S(z) = \sum_{t \in T} f_t(z).$$

Prove that

$$S'(z) = \sum_{t \in T} f'_t(z).$$

(You will have to examine the proof given in class that $S(z)$ is meromorphic, and use Weierstrass’s theorem that if $g_1, g_2, \ldots$ is a sequence of holomorphic functions on an open set $V \subset \mathbb{C}$ which converges uniformly on compact subsets of $V$ to a function $g_\infty$ then $g'_1, g'_2, \ldots$ converges to $g'_\infty$.)

3. In class we showed that

$$\left(\frac{1}{(z-n)^2}\right)_{n \in \mathbb{Z}}$$

is a LUAS family on $\mathbb{C}$. The same argument shows that

$$\left(\frac{1}{(z-n)^k}\right)_{n \in \mathbb{Z}}$$

is LUAS for any $k \geq 2$.

(a) Using the formula

$$\sum_{n \in \mathbb{Z}} \frac{1}{(z-n)^2} = \frac{\pi^2}{\sin^2(\pi z)},$$

which was proved in class, and the result of problem #2 above, derive formulae for

$$\sum_{n \in \mathbb{Z}} \frac{1}{(z-n)^3}$$

and

$$\sum_{n \in \mathbb{Z}} \frac{1}{(z-n)^4}.$$