

Math313
Homework 2, due Friday, January 29

1. Derive a formula for the sum and prove it using the method of induction:

$$\sum_{k=1}^n k^2 =$$

2. Prove by induction that the inequality

$$2^n \leq n!$$

holds for all $n \geq 2$.

3. Do #1.14 from Howie.
4. Let $f(x) = x^2 - 1$. Find $\sup_D f$, $\inf_D f$, and $\min_D f$, $\max_D f$ if the latter two exist, for $D = [-1, 1)$, and $D = (0, 2)$.
5. Show that if two functions f and g satisfy $f(x) \leq g(x)$ for all $x \in D$, then $\sup_D g \geq \sup_D f$. Show by an example that the converse to the above does not generally hold.