

Math313
Homework 7, due Friday April 23

1. Prove the fundamental theorem of calculus only under the assumption that $f' \in RI(a, b)$, i.e. if $f' \in RI(a, b)$ then

$$f(b) - f(a) = \int_a^b f'(x)dx.$$

Hint: use problem #8 from sample test 2.

2. For the function $f = 2^{-x}$ prove that $f \in RI(-1, 1)$ by definition. Compute its integral. Do the same exercise for $f(x) = \cosh(x)$ on $[0, 2]$, $f(x) = 1/x$ on $[1, 2]$.
3. Demonstrate that the following functions are RI using any available tool:

$$\begin{aligned} f(x) &= e^{-x^2}, \quad x \in [10, 30] \\ f(x) &= \cos(1/x^3), \quad x \in [-1, 1] \\ f(x) &= \begin{cases} x^2, & 0 \leq x \leq 2, \\ x + 1, & 2 < x \leq 3, \\ \sin(x), & 3 < x \leq \pi. \end{cases} \end{aligned}$$

4. Show that the function

$$f(x) = \begin{cases} x^2, & x \in \mathbf{Q}, \\ -x^2, & x \notin \mathbf{Q} \end{cases}$$

is not RI on any interval.

- 5 Do # 5.5, 5.6, 5.8, 5.10, 5.15, 5.19, 5.26, 5.27.
6. Show that if f is even and RI on $[-a, a]$, then

$$\int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx.$$

Show that if f is odd, then

$$\int_{-a}^a f(x)dx = 0.$$