Math313
Sample problems for Test 1.

1. Formulate the principle of mathematical induction.

2. State the definition of a non-Cauchy sequence.

3. State the axiom of completeness of $\mathbb{R}$.

4. Let $A = (0,1) \cup [2,4)$ and $B = \{\frac{1}{2}\} \cup (3,5]$. Find $A \cup B$, $A \cap B$, $A \backslash B$, $B \backslash A$, $A \bigtriangleup B$.

5. Let $f(x) = \sin(x)$. Determine $\sup_{[0,\pi/3)} f$, $\inf_{[0,\pi/3)} f$, $\max_{[0,\pi/3)} f$, $\min_{[0,\pi/3)} f$, if the latter two exist.

6. Find the set of all values x for which
$$\frac{x+1}{3x-1} \geq 1,$$
$$\frac{(x-1)^2(2x+3)}{(3x-2)^3 x} \leq 0.$$

7. Prove using the definition of the limit:
$$\lim_{n\to\infty} \quad \frac{n+2}{n+3} = 1$$
$$\lim_{n\to\infty} \quad 2 - \frac{(-1)^n}{n} = 2$$
$$\lim_{n\to\infty} \quad \frac{n^2}{n^3-1} = 0.$$

8. Compute the limits
$$\lim_{n\to\infty} \quad \sqrt[n]{n}$$
$$\lim_{n\to\infty} \quad \frac{2^{2/n}}{1+n}$$
$$\lim_{n\to\infty} \quad \frac{3n^2-1}{5n^2-\sqrt[n]{3}}.$$

9. Show directly that every Cauchy sequence is bounded without the use of the Cauchy theorem.

10. Show that the sequence

$$a_n = 1 - \frac{1}{2} + \frac{1}{4} - ... + (-1)^n \frac{1}{2^n}$$

has a limit. Compute it.

11. Compute the sums of the series

$$\sum_{n=4}^{\infty} \frac{(-1)^n}{3^n}; \quad \sum_{n=2}^{\infty} \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

12. Determine convergence of the series

$$\sum_{n=1}^{\infty} \frac{2^{1/n}}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{n^5 - n + 1}{n^6 + n - 1}$$

$$\sum_{n=1}^{\infty} \frac{\sqrt[n]{n}(n^3 + 1)}{\cos(1/n)(n^5 - 1)}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2 + 1}$$

$$\sum_{n=1}^{\infty} \frac{3^{2n}}{n!}$$

$$\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}.$$