

Math313
Sample problems for Test 2.

1. Review all definitions and statements of theorems from Chapters 3,4. There will be two questions on those.

2. Determine all points of discontinuity of $f(x) = \frac{\tan(x)}{1-x}$.

3. Show that f is continuous by definition

$$f(x) = \frac{x^2 - x + 1}{x + 2}, \text{ at } x = 0.$$

$$f(x) = \begin{cases} x, & x \in \mathbf{Q} \\ x^2, & x \notin \mathbf{Q} \end{cases} \text{ at } x = 1.$$

4. Prove that the Heine-Borel Theorem is not valid for the interval $(0, 1)$.

5. Prove that if $f'(a) = f''(a) = 0$ and $f'''(a) \neq 0$, then a is not a point of local extremum. Assume that the function is four times differentiable.

6. Prove that if $f'''(x) = 0$ for all x then f is a quadratic function.

7. Find all local extrema of

$$f(x) = \frac{x^2 + 1}{x^2 - 1}.$$

8. Let f be differentiable on $[a, b]$. Show that for every dissection of $[a, b]$ with points x_0, x_1, \dots, x_n there are points $c_i \in (x_{i-1}, x_i)$ such that

$$f(b) - f(a) = \sum_{i=1}^n f'(c_i)(x_i - x_{i-1}).$$