RAMSEY THEORY ON TREES AND APPLICATIONS TO INFINITE GRAPHS

NATASHA DOBRINEN

The Infinite Ramsey Theorem states that given $n, r \geq 1$ and a coloring of all n-sized subsets of $\mathbb{N}$ into r colors, there is an infinite subset of $\mathbb{N}$ in which all n-sized subsets have the same color. Extensions of Ramsey's Theorem to homogeneous structures have been studied for several decades, in particular infinite graphs. In this setting, one colors all copies of a finite graph within a given infinite graph, the goal being to find an infinite graph isomorphic to the original one in which as few colors as possible are used. Analogously to the Kechris-Pestov-Todorcevic correspondence between the Ramsey property and extreme amenability of universal minimal flows, Zucker found a correspondence has found by Zucker in [9] between Ramsey properties for infinite structures and completion flows, answering a question in [4]. This additionally motivates the search for infinite structures with good Ramsey properties.

Many results involving the Ramsey theory of infinite graphs utilize Ramsey theorems on trees. We will review theorems of Halpern-Läuchli [3] and Milliken [6], and show how these theorems were central to work of Sauer [8] and Laflamme, Sauer, and Vuksanovic [5] determining the Ramsey theory of the Rado graph. In particular, we will provide the forcing proof of the Halpern-Läuchli Theorem due to Harrington, as this sets the stage for current work. Then we will give an overview of the speaker's work on the Ramsey theory of the Henson graphs, the universal homogeneous k-clique-free graphs, in [1] and [2], answering a question implicit in [7] and [4]. The methods developed for this are finding applications to other kinds of structures, and we will conclude with some current and future directions for the developing field of Ramsey theory on infinite structures.

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Department of Mathematics, University of Denver, C.M. Knudson Hall, Room 300, 2390 S. York St., Denver, CO 80208 U.S.A.

E-mail address: natasha.dobrinen@du.edu

URL: http://web.cs.du.edu/~ndobrine