Problem 1. Prove the Chinese Remainder Theorem: If \( d_1, \ldots, d_n \) are relatively prime natural numbers, and \( a_1, \ldots, a_n \) are such that for all \( i, a_i < d_i \), then there is some \( c \), such that for all \( i, c = a_i \mod d_i \).

Recall that in class we defined a formula \( \phi^*_\text{prime}(n,p) \) in a \( \Sigma_1 \) form, such that \( \mathfrak{A} \models \phi^*_\text{prime}[n,p] \) iff \( p \) is the \( n \)-th prime. Here \( \mathfrak{A} = (\mathbb{N}, 0, S, +, \cdot, <) \) is the standard model of PA.

Problem 2. Show that \( \phi^*_\text{prime}(n,p) \) is \( \Delta_1 \) by writing a \( \Pi_1 \) formula and showing that it is equivalent to \( \phi^*_\text{prime}(n,p) \).

Problem 3. Show that any model \( \mathfrak{B} \) of PA is an end-extension of the standard model \( \mathfrak{A} \). I.e. show that there is a one-to-one function \( h : \mathbb{N} \to |\mathfrak{B}| \), such that \( h \) is homomorphism (see the definition on page 94 in the book), and for every \( b <_\mathfrak{B} c \), if \( c \in \text{ran}(h) \), then \( b \in \text{ran}(h) \).

Problem 4. Suppose that \( \phi \) is a \( \Delta_0 \)-formula, such that \( \mathfrak{A} \models \phi \). Show that any model \( \mathfrak{B} \) of PA is satisfies \( \phi \). Conclude that if \( \phi \) is a \( \Delta_0 \)-formula, then \( \mathfrak{A} \models \phi \) iff \( \text{PA} \models \phi \).