MATH 121 (9am, S.Smith)   Exam 1   Fri 17 Sept 2004

Circle your Tu/Thurs discussion time: 8   9   10   11

Circle your TA name: Katherine Bird   Ilker Yuce   Davender Sahota

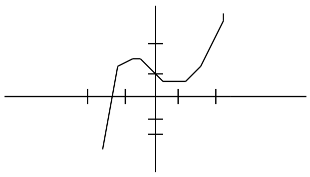
Last Name ______________ **First Name** ______________ **UIN** ______________

For full credit, give a complete explanation, not just the final answer. Give exact answers whenever possible; otherwise, give answers accurate to two decimal places. When you use a graph obtained from your calculator, sketch the graph in your answer, including the axes with a scale.

Turn in this sheet, inside your exam booklet. Also in addition to your own name, write your TA's name and your discussion time on the front page of the exam booklet.

1. (10 pts.) Sketch the graph of $f(x) = x^3 - x + 1$. (Use your calculator.)

   Use the graph to estimate solutions of $x^3 - x + 1 = 0$.

   Solutions ? Graph crosses $y = 0$ *only around* $x \cong -1.32$

2. (15 pts.) Two cars leave a gas station at the same time, one traveling north and the other south. The northbound car travels at 50 mph. After 3 hours, the cars are 345 miles apart. How fast is the southbound car traveling?

   Let s *be the southbound rate. Then the cars separate at rate* $50 + s$.

   Using distance $=$ *rate* $\times$ *time, we have* $345 = (50 + s)3$,

   from which we get $115 = 50 + s$, *and so* $\boxed{s = 65}$.

3. (15 pts.) Given $f(x) = x - x^2$, compute the difference quotient $\dfrac{f(x+h) - f(x)}{h}$, and simplify (assuming $h \neq 0$).

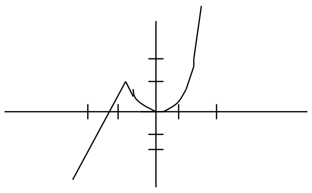
$$\frac{\Big((x+h) - (x+h)^2\Big) - \Big(x - x^2\Big)}{h} = \frac{\Big((x+h) - (x^2 + 2xh + h^2)\Big) - \Big(x - x^2\Big)}{h}$$

$$= \frac{x + h - x^2 - 2xh - h^2 - x + x^2}{h} = \frac{h - 2xh - h^2}{h} = \frac{h(1 - 2x - h)}{h}$$

$$= \boxed{1 - 2x - h}.$$

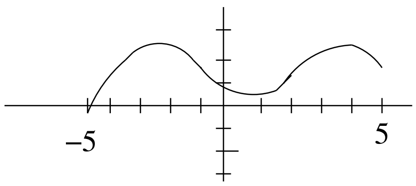
4. (15 pts.) Sketch the graph of the function $f(x) = \begin{cases} x^2 & \text{if } x \geq -1 \\ 2x+3 & \text{if } x < -1 \end{cases}$.

Then find $f(-2)$.



Then $f(-2) = 2(-2) + 3 = -1$

5. (15 pts.) In the graph of the function $f(x)$ given below:
find the approximate intervals on which it is increasing and decreasing;
and find the locations (x-values) where local maxima and minima occur.



Increasing on $(-5, -2.5)$ *and* $(1, 4)$*; decreasing on* $(-2.5, 1)$ *and* $(4, 5)$.

Local max at approx. $x = -2.5, 4$*; local min at approx.* $x = 1$.

(I had intended to EXCLUDE the end points, but did not make this clear.

So you can also include the endpoints as local minima.)

6. (15 pts.) Write the rule of a function g, whose graph can be obtained from the graph of $f(x) = x^2 + 2$, by first shifting horizontally 5 units to the left, and then shifting vertically upward by 4 units. (Do not simplify.)

Then $\boxed{g(x) = \Big((x+5)^2 + 2\Big) + 4}$.

7. (15 pts.) Find the inverse of the function $f(x) = \dfrac{x+2}{4x-3}$.

Solve for x *in terms of* y*, then convert to* x*: From* $y = \dfrac{x+2}{4x-3}$,

we get $y(4x-3) = x+2$*, so* $2+3y = 4xy - x = x(4y-1)$*, and hence* $x = \dfrac{3y+2}{4y-1}$.

So $\boxed{f^{-1}(x) = \dfrac{3x+2}{4x-1}}$.