MATH 121 (9am, S.Smith) Exam 2 Solutions Fri 15 Oct 2004

1. (10 pts.) Find the remainder when $f(x) = x^{10} + 2x^8$ is divided by $x + 1$. (Hint: you do not need to actually divide.)

   By Remainder Theorem, get $f(-1) = (-1)^{10} + 2(-1)^8 = 1 + 2 = 3$.

2. (15 pts.) Find a polynomial $f(x)$ with real coefficients, of degree 4, whose only roots are 4, $3 + i$, and $3 - i$.

   Complex roots occur in conjugate pairs, so 4 *must have multiplicity* 2.

   So $(x - 4)^2(x - (3 + i))(x - (3 - i))$ *works (can multiply by any constant).*

   This gives $(x^2 - 8x + 16)(x^2 - 6x + 10) = x^4 - 14x^3 + 74x^2 - 176x + 160$.

3. (15 pts.) Given the rational function $\frac{2 - x}{x - 3}$, find the x- and y-intercepts; find the vertical and horizontal asymptotes; and sketch a graph, labeling those features.

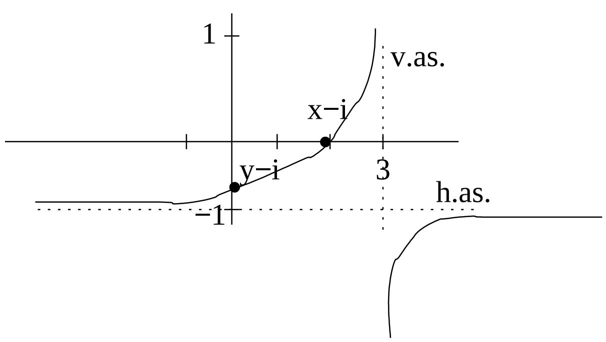
   For $x = 0$, *the* y*-intercept is the function value* $\frac{2}{-3} = -\frac{2}{3}$.

   For $y = 0$, *the* x*-intercept is the root* 2 *of the numerator* $2 - x$.

   The vertical asymptote is the root $x = 3$ *of the denominator* $x - 3$.

   For the horizontal asymptote, divide numerator and denominator by x,

   to get $\frac{\frac{2}{x} - 1}{1 - \frac{3}{x}}$; *for large values of* $|x|$, *this approaches* $\frac{-1}{1} = -1$.



4. (15 pts.) Factor $f(x) = x^3 + x^2 + x + 1$ completely over the complex numbers, given that one root is i.

   Then $-i$ *is also a root, so* $f(x)$ *is divisible by* $(x + i)(x - i) = x^2 + 1$.

   The quotient is $x + 1$, *so* $f(x) = (x + i)(x - i)(x + 1)$.

5. (15 pts.) Given $f(x) = e^{2x}$, compute the difference quotient $\frac{f(x + h) - f(x)}{h}$, and simplify (assuming $h \neq 0$).

   $$\frac{e^{2(x+h)} - e^{2x}}{h} = \frac{e^{2x}e^{2h} - e^{2x}}{h} = e^{2x} \cdot \frac{e^{2h} - 1}{h}.$$

6. (15 pts.) How long will it take to double an investment of \$ 500 at 7 % annual interest, compounded continuously?

   $1000 = 500e^{.07t}$, *so* $2 = e^{.07t}$, *so* $\ln(2) = .07t$, *and then* $t = \frac{\ln(2)}{.07} \cong 9.9021$ *years.*

7. (15 pts.) Solve $\ln(3x-5) = \ln(11) + \ln(2)$.

Exponentiate both sides: $3x - 5 = 11 \cdot 2 = 22$, *so* $3x = 27$ *and hence* $x = 9$.