MATH 121 Exam 3 (Solutions) Friday 15 November 2002 (Prof. S. Smith)

Circle your TA's name: Li Feng Weiya Zhang Brett Pontarelli

Circle your discussion time: 8 9 10 11

Last Name _____ **First Name** _____ **SSN** _____

SHOW ALL WORK. NO CREDIT for answers only

- Give exact answers whenever possible, otherwise give answers accurate to two decimal places. Sketch any calculator graph you use including the axes with a scale.
- Turn in this sheet with your test booklet. Write your TA's name and your discussion time on the booklet. BE SURE YOUR CALCULATOR IS SET TO THE RIGHT MODE, DEGREES OR RADIANS.

1. (10 pts) Give the EXACT value of the following: (can do without calculator)

   (a) $\cos(\frac{4\pi}{3})$ (b) $\csc(\frac{-\pi}{2})$

   (a) $-\frac{1}{2}$ *(b)* -1

2. (20 pts) Which of the following statements are identities? For each one, state YES or NO. If NO, give a brief explanation or a counterexample.

   (a) $\cot(x-\pi) = \cot(x)$ (b) $\cos(6\pi - t) = -\cos(t)$

   (a) YES *(b) NO (different graphs)*

   (c) $\csc^2(x) + \cot^2(x) = 1$ (d) $\cos(x) = \dfrac{\cot(x)+\cos(x)}{1+\csc(x)}$

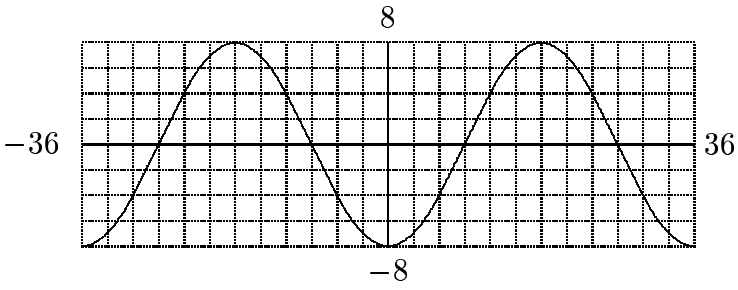
   (c) NO (different graphs) *(d) YES*

3. (20 pts) (a) Find a possible formula for the function whose graph is shown here and use your formula to answer the following questions:

   (b) What is the period?

   (c) What is the amplitude?

   (d) What is the phase shift?



Let's start with (b) ...

(b) period 36 *(so* $b = \frac{2\pi}{36} = \frac{\pi}{18}$*)* *(c) amplitude* 8 *(so* $A = 8$ *... or use* $A = -8$ *...)*

(d) To use cos*, phase shift is* $(\pm)$ *half a period* $= \pm 18$ *(so* $\pm 18 = \frac{-c}{b} = \frac{-c}{\frac{\pi}{18}}$ *so that* $c = \pm\pi$*)*

(a) So $8\cos(\frac{\pi t}{18} \pm \pi)$ *(or* $-8\cos(\frac{\pi t}{18})$ *if using* $A = -8$ *to get phase shift of* 0*)*

(or $8\sin(\frac{\pi t}{18} + \frac{3\pi}{2})$*, or* $8\sin(\frac{\pi t}{18} - \frac{\pi}{2})$*, or ...)*

4. (10 pts) If $(-3, -2)$ lies on the terminal side of an angle of t radians in standard position, find:

   (a) $\cos(t)$. (b) $\sin(t)$.

   (a) $\frac{-3}{\sqrt{13}}$ *(b)* $\frac{-2}{\sqrt{13}}$

5. (10 pts) If $\tan(t) = -\frac{6}{5}$ and $\cos(t) < 0$, find:

   (a) $\cos(t)$. (b) $\sin(t)$.

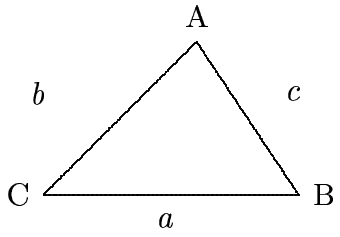
   $t = \tan^{-1}(-1.2) = -.8760$ *radians* $= -50.19$ *degrees from calculator.*

   But $\cos t < 0$*, so* $\sin t > 0$*, so quadrant II;*

   so angle is $-.88 + \pi$ *radians or* $-50.19 + 180 = 129.81$ *degrees.*

   So get (a) $-.64$ *(b)* $.77$

6. (15 pts) For the following figure, given $a = 12cm, b = 18cm$, and angle $C = 34°$,



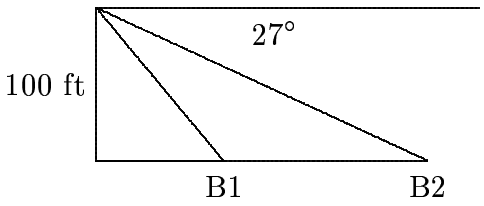
solve the triangle for side c and angles A and B (in degree). Round off answers to 2 decimal places.

(L. cosines:) $c^2 = 12^2 + 18^2 - 2 \cdot 12 \cdot 18 \cos 34° = 109.86$ *so* $c = 10.48$

(L. cosines:) $\cos B = (12^2 + 10.48^2 - 18^2)/(2 \cdot 12 \cdot 10.48) = -.279$ *so* $B = 106.2°$

and then $A = 180° - 34° - 106.2° = 39.8°$.

7. (15 pts) George is on top of a lighthouse 100 feet tall and sees two boats in the harbor on the same straight line with the lighthouse. If the angle of depression from the horizontal to $B2$ is 27°, and the angle from the horizontal to $B1$ is 46°, how far apart are the two boats?



One method: Use two right triangles, with top angles $90° - 46° = 44°$ *and* $90° - 27° = 63°$.

Then $B2/100 = \tan 63°$ *and* $B1/100 = \tan 44°$,

so $B2 - B1 = 100\ (\tan 63° - \tan 44°) = 99.69$ *ft.*