MATH 121 (9am, S.Smith) Exam 3 Solutions Fri 12 Nov 2004

1. (10 pts.) Give *exact* values for $\cos(\frac{13\pi}{6})$ and $\sin(-\frac{\pi}{4})$.

   Use period, negative-angle identities and "special angles": Then

   $\cos(\frac{13\pi}{6}) = \cos(2\pi + \frac{\pi}{6}) = \cos(\frac{\pi}{6}) = \boxed{\frac{\sqrt{3}}{2}}$ *and* $\sin(-\frac{\pi}{4}) = -\sin(\frac{\pi}{4}) = \boxed{-\frac{\sqrt{2}}{2}}$.

2. (15 pts.) Given $\sin t = \frac{12}{13}$ and the condition $\frac{\pi}{2} < t < \pi$, find $\cos t$.

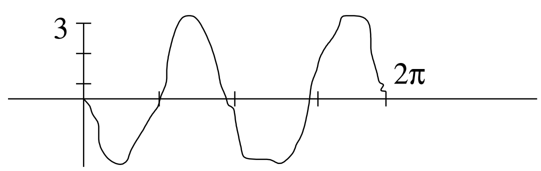
   By Pythagorean identity, $\sin^2 t + \cos^2 t = 1$.

   So $(\frac{12}{13})^2 + \cos^2 t = 1$, *giving* $\cos^2 t = 1 - \frac{144}{169} = \frac{25}{169}$, *so that* $\cos t = \pm\sqrt{\frac{25}{169}} = \pm\frac{5}{13}$.

   Since $\frac{\pi}{2} < t < \pi$, *the cosine is negative, so* $\cos t = \boxed{-\frac{5}{13}}$.

3. (15 pts.) State the amplitude, period, and phase shift of $f(t) = 3\cos(2t + \frac{\pi}{2})$. Sketch the graph of this function.

   Amplitude $\boxed{A = 3}$; $b = 2$ *so period* $\frac{2\pi}{b} = \frac{2\pi}{2} = \boxed{\pi}$; $c = \frac{\pi}{2}$, *so phase shift* $\boxed{-\frac{c}{b} = -\frac{\pi}{4}}$.



4. (15 pts.) If $\sin t = .6$ and $\cos t = .8$, give *exact* values of $\sin(t + \frac{\pi}{3})$ and $\cos 2t$. (Use identities, and you need not simplify.)

   $\sin(t + \frac{\pi}{3}) = \sin t \cos(\frac{\pi}{3}) + \cos t \sin(\frac{\pi}{3}) = .6(\frac{1}{2}) + .8(\frac{\sqrt{3}}{2}) = \boxed{.3 + .4\sqrt{3}}$.

   $\cos 2t = \cos^2 t - \sin^2 t = (.8)^2 - (.6)^2 = .64 - .36 = \boxed{.28}$.

5. (15 pts.) Is "$(\sin x + \cos x)^2 - 1 = \sin 2x$" an identity ? (Why/why not?)

   $\boxed{\textit{Yes}}$: *the left side multiplies out to* $\sin^2 x + \cos^2 x + 2\sin x\cos x - 1$,

   which using the Pythagorean identity is $1 + 2\sin x\cos x - 1 = 2\sin x\cos x$,

   and so equals the right side $\sin 2x$ *using the double-angle identity.*

6. (15 pts.) Give the *exact* value of $\sin^{-1}(-\frac{\sqrt{3}}{2})$.

Give at least one solution of $\sin 3x = -\frac{\sqrt{3}}{2}$. (5 pts. extra: What are all solutions?)

The range of $\sin^{-1}$ *(the part of the domain of* $\sin$ *that we use) is from* $-\frac{\pi}{2}$ *to* $\frac{\pi}{2}$*, so* $\sin^{-1}(-\frac{\sqrt{3}}{2}) = \boxed{-\frac{\pi}{3}}$.

To get one solution, we apply $\sin^{-1}$ *to both sides of the equation to get* $3x = \sin^{-1}(-\frac{\sqrt{3}}{2}) = -\frac{\pi}{3}$*, so* $x = \boxed{-\frac{\pi}{9}}$.

(But note that $-\frac{2\pi}{3} = \frac{4\pi}{3}$ *is another angle whose sine is* $-\frac{\sqrt{3}}{2}$*, giving another solution* $x = -\frac{2\pi}{9}$*; and we get further solutions by adding to either of these any whole-number multiple of* $\frac{2\pi}{3}$*.)*

7. (15 pts.) (a) Solve $\ln(x+8) - \ln x = 1$.

Exponentiate both sides: $\frac{x+8}{x} = e^1$*, or* $x+8 = ex$*; so* $(e-1)x = 8$ *and* $x = \boxed{\frac{8}{e-1}}$.

(b) Give a degree-4 polynomial with real coefficients and roots $1+i$ and $2-i$.

Can use (any constant multiple of) $(x-(1+i))(x-(1-i))(x-(2-i))(x-(2+i))$

$= \boxed{(x^2-2x+2)(x^2-4x+5)} = (x^4 - 6x^3 + 15x^2 - 18x + 10)$.