Final Exam

December 12, 2002

SSN: \_\_\_\_\_

CIRCLE your lecturer/lecture time; and your TA/discussion time:

Doyle 8	Smith 9	<b>Liu</b> 11	Radford 1	Colman 2
B.Jacobs 8 9 10	L.Feng 8 9 11	C.X. Lu 11 12 1	A.Shaker 12 2	Q. Liang 1 2 3
P.Manilou 8 9 11	W. Zhang 8 9	C.L. Yan 11 12 1	K.Bird 1	A.Hallman 1 2 3
	B Pontarelli 10			

## READ THESE INSTRUCTIONS:

RETURN this exam sheet, inside your exam booklet when you hand it in.

Write all work and ANSWERS in your exam booklet (NOT on these sheets of exam questions).

If you need an additional exam booklet, ask a proctor.

There are 10 problems, each worth 20 points.

Give complete explanations, not just answers, for full credit.

Give exact answers whenever possible; otherwise give them to three decimal places.

Follow the directions for the use of a calculator if given for any problem. Sketch any calculator graph you use including the axes with a scale.

No notes or other papers are allowed during the exam.

Receiving or giving aid in a final examination is a cause for dismissal from the University.

For grading only; **DO NOT WRITE** here:

1	2	3	4	5	6	7	8	9	10	Total

**Problem 1.** Use algebra to find the inverse function of  $f(x) = 6 + \sqrt{5x - 7}$ .

**Problem 2.** Find all roots (real and complex) of  $f(x) = x^3 + 2x - 3$ , and write f(x) as a product of linear (degree 1) factors.

**Problem 3.** The height of a projectile in feet is given by  $h(t) = -16t^2 + v_0t + h_0$ , where  $v_0$  is the initial velocity, and  $h_0$  is the initial height. If the projectile is a ball is thrown upward from the top of a building 500 feet high, at a velocity of 10 feet per second,

- (a) What maximum height does the ball reach?
- (b) After how many seconds does the ball reach the ground? (It misses the building on the way down).

**Problem 4.** Solve algebraically (you may use a calculator to check your solution):

- (a)  $2\sin^2(x) + \cos(x) = 2$  (ALL solutions).
- (b)  $\log(\sqrt{x^2 10}) = 3$ .

**Problem 5.** (a) Assume that \$600 is invested at an interest rate of 8% per year. What is the amount after 9 years, with quarterly compounding?

(b) What annual interest rate (with continuous compounding) is required in order for an investment of P dollars to double after exactly 15 years?

**Problem 6.** Let  $f(x) = \frac{5x+3}{17x-9}$ .

- (a) Find the domain of f(x).
- (b) Find the horizontal asymptotes to the graph of y = f(x).
- (c) Find the vertical asymptotes to the graph of y = f(x).
- (d) Find the points where the graph of y = f(x) crosses the x-axis.
- (e) Find the points where the graph of y = f(x) crosses the y-axis.
- (f) Using (a)–(e), sketch the graph of y = f(x). Label the asymptotes and points you have identified.

**Problem 7.** Which of the following statements are identities? In each case answer yes or no. If your answer is no, find a specific x for which the statement is false.

- (a)  $\cot(x + \pi) = -\cot(x)$ .
- (b)  $e^{x+5} = e^x + e^5$ .
- (c)  $1 2\sin^2(x) = \cos(2x)$ .
- (d)  $\ln(x + 10) = (\ln x)(\ln 10)$ .
- (e)  $\arccos(\cos x) = x$ .

**Problem 8.** Given that  $tan(t) = -\frac{7}{6}$ , and  $\pi < t < 2\pi$ , find the **exact value** of the following:

(a)  $\cos(t)$ , (b)  $\csc(t)$ , (c)  $\sin(t/2)$  and (d)  $\cos(2t)$ .

(Examples of exact values:  $\frac{1}{2}$ ,  $\frac{\sqrt{7}}{10}$ ; not decimal approximations such as 0.571.)

**Problem 9.** George and Georgette are 120 feet apart on level ground looking at a very tall streetlight **between** them. The angle of inclination from George's feet to the top of the streetlight is  $27^{\circ}$  and the angle of inclination from Georgette's feet to the top of the streetlight is  $61^{\circ}$ .

- (a) How tall is the streetlight?
- (b) How far is Georgette from the base of the streetlight? [Hint: Draw a picture!]

**Problem 10.** (a) Write  $1 + \sqrt{3}i$  in polar form  $r(\cos \theta + i \sin \theta)$ , with  $0 \le \theta < 2\pi$ .

(b) Use your answer in (a) to calculate  $(1+\sqrt{3}i)^7$  in polar form.

The values of r and  $\theta$  must be expressed as **exact values**, for example  $\frac{1}{2}$ ,  $\frac{\pi}{19}$ ,  $\frac{\sqrt{7}}{10}$ , and not as decimal approximation such as 0.571.