

An Invitation to Higher Mathematics

Math 215, Fall Semester, 2001

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Sample Hour Exam 2– November 12

1. Suppose that A is a finite set with 3 elements, and B is a finite set with 5 elements. What is the cardinality of the set
 - a. $A \times B$
 - b. A^B
 - c. B^A
 - d. $\mathcal{P}(B)$
 - e. injections from A to B , $\text{Inj}(A, B)$
 - f. surjections from B to A , $\text{Sur}(B, A)$
 - g. bijections, $\text{Bij}(B, B)$
2. Suppose that $f: A \rightarrow B$ and $g: B \rightarrow C$ are both surjections. Prove that the composition $g \circ f$ is a surjection.
3. The students in a math class were given a choice of adding any combination of 3 ingredients to a pizza. Their choices were sausage, olives, or calamari (squid). 10 wanted olives, 14 wanted sausage and 12 wanted calamari. Of these, 3 wanted sausage and olives, 4 wanted sausage and calamari, 2 wanted olives and calamari, and 2 wanted all three ingredients. How many students were in the class? (Assume that everyone made some choice for a topping.)
4. Prove that there are at least two ways to choose three distinct integers between 1 and 10 whose sums are the same. (Show there are choices of integers $1 \leq n_1 < n_2 < n_3 \leq 10$ and $1 \leq m_1 < m_2 < m_3 \leq 10$ with $n_1 + n_2 + n_3 = m_1 + m_2 + m_3$ and $n_1 \neq m_1$ or $n_2 \neq m_2$ or $n_3 \neq m_3$.)
5. What is the coefficient of x^2 in the expansion of $(x - 1)^{11}$?