

An Invitation to Higher Mathematics

Math 215, Fall Semester, 2001

Problems & Exercises

Week 1 – August 20-24

1. Construct truth tables for the statements
 - (i) (not P) or (not Q)
 - (ii) $(\sim P) \wedge (\sim Q)$
 - (iii) $A \wedge (B \vee C)$
 - (iv) $(\sim P) \implies Q$
2. Write $(\sim P) \wedge (\sim Q)$ as an expression using only the operations $\sim$ and $\vee$.
3. (*turn in Monday August 27*)
Use truth tables to prove that $P \implies Q \equiv (\sim Q) \implies (\sim P)$.
4. (i) Give the definition of $|a|$ using logical cases (if $a < 0$ then ... else ...).
(ii) Prove that $|a|^2 = a^2$ for every real number a .
5. (*turn in Monday August 27*)
Prove that the square of an even integer is even.
6. (*turn in Monday August 27*)
Prove that if $a < b$ and $c < d$ then $a + c < b + d$.

Some things to ponder

Here's a little something from Lewis Carroll...

Q: You have a box containing two balls, each either black or white (but you don't know which). What color are the balls?

A: One white, one black.

Proof: Pick a ball out of the box. There is a fifty-fifty chance that it's black.

Now, if both balls had been black, the chance would have been 100%, not 50%.

If both balls were white, the chance would have been 0%.

So there must be one of each. QED.

...and a math joke:

Cat Theorem: A cat has nine tails.

Proof: No cat has eight tails.

A cat has one tail more than no cat.

Therefore, a cat has nine tails. QED.