An Invitation to Higher Mathematics
Math 215, Fall Semester, 2001

Solutions to Problems & Exercises
Week 1 – August 20-24

1. Construct truth tables for the statements
(i) (not P) or (not Q) *Solution:*

P	Q	$(\sim P) \ \vee \ (\sim Q)$
T	T	F
T	F	T
F	T	T
F	F	T

(ii) $(\sim P) \wedge (\sim Q)$ *Solution:*

P	Q	$(\sim P) \wedge (\sim Q)$
T	T	F
T	F	F
F	T	F
F	F	T

(iii) A ∧ (B ∨ C) *Solution:*

A	B	C	(B ∨ C)	A ∧ (B ∨ C)
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	F
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

(iv) $(\sim P) \Longrightarrow Q$ *Solution:*

P	Q	$(\sim P) \Longrightarrow Q$
T	T	T
T	F	T
F	T	T
F	F	F

2. Write $(\sim P) \wedge (\sim Q)$ as an expression using only the operations $\sim$ and $\vee$.
Solution:

$$(\sim P) \wedge (\sim Q) = \sim \{\sim \{(\sim P) \wedge (\sim Q)\}\} = \sim \{(\sim\sim P) \vee (\sim\sim Q)\} = \sim \{P \vee Q\}$$

3. (*turn in Monday August 27*)
Use truth tables to prove that $P \Longrightarrow Q \equiv (\sim Q) \Longrightarrow (\sim P)$.
Solution: Compare the two tables

P	Q	$P \Longrightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$(\sim Q) \Longrightarrow (\sim P)$
T	T	T
T	F	F
F	T	T
F	F	T

Since the last columns are identical, the two statements are equivalent.

4. (i) Give the definition of $|a|$ using logical cases (if $a < 0$ then ... else ...).
Solution: For a real number a, either $a > 0$, $a = 0$ or $a < 0$.
If $a \geq 0$ then define $|a| = a$, else if $a < 0$ then define $|a| = -a$.

(ii) Prove that $|a|^2 = a^2$ for every real number a.
Proof: For a real number a, either $a > 0$, $a = 0$ or $a < 0$.
If $a \geq 0$ then $|a| = a$, hence $|a|^2 = a^2$.
If $a < 0$ then $|a| = -a$, hence $|a|^2 = (-a)^2 = a^2$. $\square$

5. (*turn in Monday August 27*)
Prove that the square of an even integer is even.
Proof: An integer n is even means that $n = 2p$ for an integer p.
This implies $n^2 = (2p)^2 = 2 \cdot 2p^2$.
Thus, n^2 is twice the integer $2p^2$, so is even. $\square$

6. (*turn in Monday August 27*)
Prove that if $a < b$ and $c < d$ then $a + c < b + d$.
Proof: Use the addition rule of inequalities to obtain
$a < b \Longrightarrow a + c < b + c$
$c < d \Longrightarrow c + b < d + b$
Use the transitive rule for inequalities to obtain
$a + c < b + c = c + b < d + b = b + d$. $\square$