An Invitation to Higher Mathematics
Math 215, Fall Semester, 2001

Problems & Exercises
Week 2 – August 27-31

7. Prove by contradiction that there do not exisat integers m and n such that

$$14m + 21n = 100$$

(This is problem 4.1 of the text)

8. Prove by contradiction that for any integer n,

$$n^2 \text{ is odd} \quad \Longrightarrow n \text{ is odd}$$

(This is problem 4.2 of the text)

9. (*turn in Wednesday, September 5*)
Prove by contradiction that for any integer n,

$$n^2 \text{ is even} \quad \Longrightarrow n \text{ is even}$$

(This is problem 7 on page 54 of the text.) Hint: you may suppose that an integer n is odd if and only if $n = 2q + 1$ for some integer q.

10. (*turn in Wednesday, September 5*)
Prove that, for all real numbers a and b,

$$|a + b| \leq |a| + |b|$$

(This is problem 4.7 of the text) Hint: use cases.

11. Prove the following statements concerning positive integers a, b, and c.
(i) (a divides b) and (a divides c) $\Longrightarrow$ a divides $(b + c)$
(ii) (a divides b) or (a divides c) $\Longrightarrow$ a divides (bc)
(This is problem 4 on page 53 of the text.)

12. Which of the following conditions are necessary for the positive integer n to be divisible by 6 (proof not necessary)?
(i) 3 divides n
(ii) 9 divides n
(ii) 12 divides n
(iv) $n = 12$
(v) 6 divides n^2
(vi) 2 divides n and 3 divides n
(vi) 2 divides n or 3 divides n
Which of these conidtions are *sufficient*? (This is problem 5 on page 53 of the text.)

13. (*turn in Wednesday, September 5*)
Prove by induction on n that, for all positive integers n, $n^3 - n$ is dividible by 3. (This is problem 5.1 of the text)