An Invitation to Higher Mathematics
Math 215, Fall Semester, 2001

Problems & Exercises
Week 5 – September 17–21

19. Prove that
    i) $\{x \in \mathbb{R} \mid x^2 + x - 2 = 0\} = \{-2, 1\}$
    ii) $\{x \in \mathbb{R} \mid x^2 + x - 2 < 0\} = (-2, 1)$
    iii) $\{x \in \mathbb{R} \mid x^2 + x - 2 > 0\} = \{x \in \mathbb{R} \mid x < -2\} \cup \{x \in \mathbb{R} \mid x > 1\}$

20. Find predicates which determine the following subsets of the set of integers $\mathbb{Z}$
    i) $\{3\}$
    ii) $\{1, 2, 3\}$
    iii) $\{1, 3\}$ (*turn in this case on Monday, September 24*)

21. (*turn in Monday, September 24*)
    By using a truth table prove that

    $$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

    Draw a Venn diagram to illustrate the proof.

22. (*turn in Monday, September 24*)
    By using a truth table prove that

    $$A \subset B \iff A \cup B = B$$

23. (*turn in Monday, September 24*)
    Prove by contradiction or otherwise that

    $$A \cap B = A \cap C \ \& \ A \cup B = A \cup C \iff B = C$$

24. (*turn in Monday, September 24*)
    Using the fact that an implication is equivalent to its contrapositive, prove that, for subsets of a universal set U,

    $$A \subset B \iff B^c \subset A^c$$