An Invitation to Higher Mathematics

Math 215, Fall Semester, 2001

Problems & Exercises

Week 6 – September 24–28

"In mathematics you don't understand things, you just get used to them."

John von Neumann

25. Prove the following

$$(\exists q \in \mathbb{Z}, n = 2q + 1) \Longrightarrow \big(\exists p \in \mathbb{Z}, n^2 = 2p + 1\big)$$

26. (*turn in Monday, October 1*)
Write the following statement in terms of quantifiers and prove it.
For integers a and b, if a and b are even then so is $a + b$.

27. (*turn in Monday, October 1*)
For sets A, B, C and D prove that

$$(A \times B) \cup (C \times D) \subset (A \cup C) \times (B \cup D)$$

and give an example to show thatthese sets are not always equal.

28. (*turn in Monday, October 1*)
Define functions $f, g: \mathbb{R} \to \mathbb{R}$ by $f(x) = x^3$ and $g(x) = 1 - x$.
a) Find the functions
i) $f \circ f$
ii) $f \circ g$
iii) $g \circ f$
iv) $g \circ g$

b) List the elements of the set $\{x \in \mathbb{R} \mid fg(x) = gf(x)\}$.

29. (*turn in Monday, October 1*)
Define the composition of the function $f: X \to Y$ and the function $g: Y \to Z$ to be the function $g \circ f: X \to Z$ with $g \circ f(x) = g(f(x))$ for all $x \in X$. Prove that:
a) If f is injective and g is injective, then $g \circ f$ is injective.
b) If f is surjective and g is surjective, then $g \circ f$ is surjective.