An Invitation to Higher Mathematics

Math 215, Fall Semester, 2001

Problems & Exercises

Week 9 – October 15 –19

34. (*turn in Monday, October 22*)
For $f: X \to Y$ and $g: Y \to Z$ show that:
a) if $g \circ f$ is injective, then f is injective.
b) if $g \circ f$ is surjective, then g is surjective.

35. (*turn in Monday, October 22*)
For $f: X \to Y$ and $g: Y \to X$, use problem 34 to prove that if $g \circ f$ is injective and $f \circ g$ is surjective, then f is bijective.

36. (*turn in Monday, October 22*)
For $f: X \to Y$ and $g: Y \to X$, if $g \circ f$ and $f \circ g$ are both bijective, show that f and g are both bijective.

37. (*turn in Monday, October 22*)
The functions $f: X \to Y$ and $g: Y \to X$ are inverses of each other if $\forall x \in X,\ g(f(x)) = x$, and $\forall y \in Y,\ f(g(y)) = y$. Use problem 36 to prove that f and g are both bijections.

38. (*turn in Monday, October 22*)
Given five points in the plane with integer coordinates, prove that the midpoint between at least one of pair of the points has *integer* coordinates.

39. (*turn in Monday, October 22*)
Let $S \subset \{1, 2, 3, \dots, 2n\}$ where S has $n+1$ elements. Show that S contains two numbers such that one divides the other.

    Recall that $n|m$ means there is some integer p so that $m = p \cdot n$.

    Hint: Any number m can be written uniquely as an odd number times a power of 2, or $m = (2k-1) \cdot 2^j$. Then consider the function $f: S \to \{1, 2, 3, \dots, n\}$ defined by $f(m) = k$.