An Invitation to Higher Mathematics

Math 215, Fall Semester, 2001

Problems & Exercises Week 10 – October 22–26

40. (turn in Wednesday, October 31)

Show that the set of all positive multiple of 3 is a countable set. (Your answer should be a bijection from \mathbb{N} to this subset of the positive integers.)

41. (turn in Wednesday, October 31)

Let A be a countably infinite set with bijection $g: \mathbb{N} \to A$. Suppose that B is a finite set with $A \cap B = \emptyset$. Show that $A \cup B$ is countably infinite. (Your answer should use the function g and a counting of the set B to produce a bijection from \mathbb{N} to $A \cup B$.)

42. (turn in Wednesday, October 31)

Let A and B be a countable sets, which need not be disjoint. Show that $A \cup B$ is countable. (You can assume the result we proved in class that a subset of a countable set is countable.)

43. (turn in Wednesday, October 31)

Show that the product $\mathbb{N} \times \mathbb{N}$ is countable. (Your answer should be a bijection $f: \mathbb{N} \to \mathbb{N} \times \mathbb{N}$. Hint: you may want to read the section 3.5 of the handout on Functions.)