An Invitation to Higher Mathematics
Math 215, Fall Semester, 2001

Problems & Exercises
Week 14 – November 19–21

56. (*turn in Wednesday, November 28*)
For a and b as follows, write down q and r such that $a = bq + r$ and $0 \leq r < b$.
(i) a = 100, b = 3;
(ii) a = 3, b = 100;
(iii) a = 100, b = 7;
(iv) a = -100, b = 7;
(v) a = -105, b = 7;
(vi) a = 7684, b = 4148;
(vii) a = 1234567, b = 1357;
(viii) a = 0, b = 17;

57. (*turn in Wednesday, November 28*)
Prove that, for an integer a, a^2 is divisible by 3 if and only if a is divisible by 3.

58. (*turn in Wednesday, November 28*)
Prove that if n is a perfect square then $n = 4q$ or $n = 4q + 1$ for some $q \in \mathbb{Z}$. Deduce that 1234567 is not a perfect square.

59. (*turn in Wednesday, November 28*)
Use the Euclidean algorithm to find the greatest common divisor of 7684 and 4148.

60. (*turn in Wednesday, November 28*)
Find integers m and n such that $68 = 7684m + 4148n$.

61. Use the Euclidean algorithm to find the greatest common divisor of 11033442 and 1102246.

62. Find integers m and n such that $578 = 11033442m + 1102246n$.