Math 215: **Homework for week 13, with new problems 44a,44b**
Prof. S. Smith due Monday 15 April 2002

Replace problem 44 (on the original assignment) with the following two problems, based on 2 and 4 of Exam 2 given Fri 5 April:

Problem 44a: Decide if the following functions are injective and/or surjective—by giving a proof:
(i) $f : \mathbf{R}^2 \to \mathbf{R^2}$ defined by $f(x_1, x_2) = (x_2, 2x_2)$.
(ii) $f : \mathbf{R}^2 \to \mathbf{R^2}$ defined by $f(x_1, x_2) = (x_1 + x_2, x_1 - x_2)$.

Problem 44b: Show that the following sets are denumerable (countably infinite), by giving a bijection with (possibly a subset of) $\mathbf{Z}^m$ or $\mathbf{N}^m$ for a suitable $m > 1$:
(i) the set of "Pythagorean triples": that is, all (a, b, c) with $a, b, c \in \mathbf{N}$ such that $a^2 + b^2 = c^2$. (One example is the triple $3, 4, 5$).
(ii) the set of integer 2×2 symmetric matrices: that is, all $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ with $a, b, c \in \mathbf{Z}$. (One example is the matrix $\begin{pmatrix} 5 & 8 \\ 8 & -12 \end{pmatrix}$

Problem 45: (essentially 12.1 in the text)
Write down the first 10 rows of Pascal's triangle. use this to find the coefficient of a^7b^3 in $(a + b)^{10}$.

Problem 46: (Similar to 12.2 in the text)
Find the coefficient of x^4 in $(x + 2)^7$.

Problem 47: (12.4 in the text)
Three people each select a main dish from a menu of 5 items. How many choices are possible
(i) if we record who selected which dish (as the waiter should)
(ii) if we ignore who selected which dish (as the chef could) ?

Problem 48: (12.6 in the text)
Prove that the product of any n consecutive positive integers is divisible by $n!$.

Problem 49: (p. 185, no. 20)
Use the pigeonhole principle to prove that, given a set S of 10 distinct positive integers less than 107, there exist two disjoint subsets of S with the same sum.