Math 310: **Hour Exam 1** (Solutions)
Prof. S. Smith: Mon 29 Sept 1997

Problem 1: Find the row-reduced echelon form of the following augmented matrix. INDICATE the row operations you use.

$$(A|b) = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 4 & 5 & 8 \end{array}\right).$$

Then give the solutions of the corresponding linear system $Ax = b$.

$$\stackrel{A_2^{-1\times 1}, A_3^{-3\times 1}}{\rightarrow} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & -1 & -2 & -2 \\ 0 & -2 & -4 & -4 \end{array}\right) \stackrel{M^{-1}\times 2}{\rightarrow} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 2 \\ 0 & -2 & -4 & -4 \end{array}\right) \stackrel{A_1^{-2\times 2}, A_3^{2\times 2}}{\rightarrow} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

So solutions are $(r, 2-2r, r)$ *for free variable* $x_3 = r$.

Problem 2: Give the LU-decomposition of

$$A = \begin{pmatrix} 4 & 5 \\ 2 & 3 \end{pmatrix};$$

that is, find lower-triangular L and upper-triangular U, so that $A = LU$.

Get U *from* $\stackrel{A_2^{-\frac{1}{2}\times 1}}{\rightarrow} \begin{pmatrix} 4 & 5 \\ 0 & \frac{1}{2} \end{pmatrix}$. *So* L *from inverse operation* $A_2^{+\frac{1}{2}\times 1}$ *is* $\begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix}$

Problem 3: Find the inverse (any method) of:

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix}.$$

Quick via adjoint: det (by bottom row) is $0 - 0 + 2(-2) = -4$

so inverse is $-\frac{1}{4}\begin{pmatrix} -1 & -3 & 2 \\ -2 & 2 & 0 \\ 1 & -1 & -2 \end{pmatrix}$

Problem 4: Find the determinant of

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 3 & 4 & 2 \\ 1 & 3 & 1 \end{pmatrix}.$$

What is the determinant of A^{-1} ?

Top row: $1(4.1 - 3.2) - 1(3.1 - 1.2) = (-2) - (1) = -3$. *Then* $\det(A^{-1}) = \frac{1}{\det(A)} = -\frac{1}{3}$.

Problem 5: (a) Is $(2, 5, 4)$ a linear combination of $(1, 1, 2)$ and $(1, 3, 2)$?
Either give coefficients, or explain why not possible.

No: Set up augmented matrix $(A|b) = \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 1 & 3 & 5 \\ 2 & 2 & 4 \end{array}\right)$.

Row-reduction quickly produces coefficients $\frac{1}{2}$ *and* $\frac{3}{2}$.

(b) Let V be the space of 2×2 matrices, and W the subSET of lower triangular matrices. Show that the W is a subSPACE of V.

$A \in W$ *says lower triangular, meaning* $A_{2,1} = 0$, *so* A *has form* $\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}$.

Similarly if $B \in W$ *it has form* $\begin{pmatrix} d & 0 \\ e & f \end{pmatrix}$

Is $A + B \in W$*? It is* $\begin{pmatrix} a+d & 0 \\ b+e & c+f \end{pmatrix}$, *also lower-triangular, so yes.*

For scalar r, *is* $rA \in W$*? It is* $\begin{pmatrix} ra & 0 \\ rb & rc \end{pmatrix}$, *also lower-triangular, so yes.*