Math 310: **Hour Exam 1** (Solutions)
Prof. S. Smith: Mon 28 Sept 1998

Problem 1: (a) Using Gauss-Jordan, find the row-reduced echelon form of the following augmented matrix. (INDICATE the row operations you use).

$$(A|b) = \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 2 & 3 & 4 & 9 \end{array}\right).$$

$$\overset{A_3^{-2\times 1}}{\rightarrow} \left(\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & 1 & 2 & 3 \\ 0 & -1 & -2 & -3 \end{array}\right) \overset{A_1^{-2\times 2}, A_3^{1\times 2}}{\rightarrow} \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

(b) Then give the solutions of the corresponding linear system $Ax = b$.
So solutions are $(r, 3-2r, r)$ *for free variable* $x_3 = r$.

Problem 2: (a) What elementary row OPERATION will change

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \text{ to } B = \begin{pmatrix} 1 & 2 \\ 0 & -3 \end{pmatrix} \ ?$$

$A_2^{-2\times 1}$ *: add* -2 *times 1st row to 2nd.*
(b) What elementary row MATRIX E will, by left muliplication, perform the same operation? (that is, $EA = B$)
$E = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$ *: (obtained by doing that operation to the identity matrix).*

Problem 3: (a) Find the inverse (any method) of:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

Using $(A|I)$ *method: use* $A_2^{-1\times 1}, A_3^{-1\times 1}$ *to clear first column; then* $A_3^{-1\times 2}$ *to clear second column.*
Get inverse: $\begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \end{pmatrix}$
(b) Give the LU-decomposition of

$$A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix};$$

that is, find lower-triangular L and upper-triangular U, so that $A = LU$.
Apply $A_2^{-1\times 1}$ *to get* $U = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. *So* L *from inverse operation* $A_2^{+1\times 1}$ *is* $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

Problem 4: (a) Find the determinant of

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

Top row: $1(1.1-0.1)-1(0.1-1.1)+0=(1)-(-1)=2.$

(b) Use Cramer's rule to solve

$$\left(\begin{array}{cc|c} 1 & 2 & 4 \\ 3 & 4 & 10 \end{array}\right)$$

$\det(A)=1.4-3.2=-2$, *so* $x_1=-\frac{1}{2}(4.4-10.2)=2$ *and* $x_2=-\frac{1}{2}(1.10-3.4)=1.$

Problem 5: (a) Is $(1,0,1)$ in the span of $(1,1,1)$ and $(1,2,1)$?
Either give coefficients in a linear combination, or explain why it is not possible.

Yes: Set up augmented matrix $(A|b) = \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{array}\right).$

Row-reduction quickly produces coefficients 2 *and* -1.

(b) Let V be the space of 2×2 matrices, and W the subSET of diagonal matrices.
Show that W is a subSPACE of V.

$A \in W$ *says diagonal, meaning* 0 *off the diagonal, so* A *has form* $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}.$

Similarly if $B \in W$ *it has form* $\begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix}$

Is $A+B\in W$*? It is* $\begin{pmatrix} a+c & 0 \\ 0 & b+d \end{pmatrix}$, *also diagonal, so yes.*

For scalar r, *is* $rA \in W$*? It is* $\begin{pmatrix} ra & 0 \\ 0 & rb \end{pmatrix}$, *also diagonal, so yes.*

We see "yes", W *is a subspace of* V.