Math 310: **Hour Exam 1** (Solutions)

Prof. S. Smith: Fall 1995

Problem 1: (a) Find the row-reduced echelon form of

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

$$\stackrel{A_2^{-4\times 1}, A_3^{-7\times 1}}{\to} \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{pmatrix} \stackrel{M_{-\frac{1}{3}\times 2}}{\to} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{pmatrix} \stackrel{A_1^{-2\times 2}, A_3^{6\times 2}}{\to} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

(b) What are the solutions of the system $Ax = 0$? (Check!)

Third variable is free, so solutions $x_3(1, -2, 1)^T$.

Problem 2: Give the LU-decomposition of

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix};$$

that is, find lower-triangular L and upper-triangular U, so that $A = LU$.

Get U *from* $\stackrel{A_2^{-1\times 1}}{\to} \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$. *So* L *from inverse operation* $A_2^{+1\times 1} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

Problem 3: Use Cramer's rule (determinants) to solve $Ax = b$ given by

$$(A|b) = \left(\begin{array}{cc|c} 1 & 1 & 5 \\ 1 & 2 & 7 \end{array}\right).$$

First $\det \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = 1$, *so* $x_1 = (\frac{1}{1}) \det \begin{pmatrix} 5 & 1 \\ 7 & 2 \end{pmatrix} = 3$ *and* $x_2 = \det \begin{pmatrix} 1 & 5 \\ 1 & 7 \end{pmatrix} = 2.$

Problem 4: (a) Find the inverse (by any method) of

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}.$$

Quick via adjoint: $\det \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix} = -1$, *so inverse is* $-\begin{pmatrix} 5 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ 3 & -1 \end{pmatrix}$.

(b) Use the above to express the solutions of $Ax = b$ in terms of the constants b_1 and b_2.

By $A^{-1}b$, *namely* $x_1 = -5b_1 + 2b_2$ *and* $x_2 = 3b_1 - b_2$.

Problem 5: (a) Is $(1,2,3)$ in the span of $(4,0,5)$ and $(6,0,7)$?

No—for example, any linear combination of the latter two vectors has 0 in second entry.

(b) Let V be the space of all functions (with at least 2 derivatives).
Let W be the subSET of all functions f which are solutions of the differential equation

$$f'' + 5f = 0.$$

Show that the solution set W is a subSPACE of V.

Take $f, g \in W$, and scalar c: Then we have $f'' + 5f = 0 = g'' + 5g$.

Is $f + g \in W$? $(f+g)'' + 5(f+g) = (f'' + 5f) + (g'' + 5g) = 0 + 0 = 0$, so OK.

Is $cf \in W$? $(cf)'' + 5(cf) = c(f'' + 5f) = c(0) = 0$, also OK.