Math 310: **PRACTICE Hour Exam 1** (Solutions)

Prof. S. Smith: (Fall 94: orig. given 27 Sept 1993)

Problem 1: Find the general solution of $\begin{pmatrix} 1 & 3 & 3 \\ 2 & 6 & 9 \\ -1 & -3 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 5 \end{pmatrix}$.

$$\overset{A_2^{-2\times 1}, A_3^{1\times 1}}{\rightarrow} \left(\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 6 & 6 \end{array}\right) \overset{M_{\frac{1}{3}\times 2}}{\rightarrow} \left(\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 6 & 6 \end{array}\right) \overset{A_1^{-3\times 2}, A_3^{-6\times 2}}{\rightarrow} \left(\begin{array}{ccc|c} 1 & 3 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Thus x_2 is free variable; with $x_2 = 1$ in $Ax = 0$, special solution is $(-3\,1\,0)$.

With $x_2 = 0$ in $Ax = b$, particular solution is $(-2\,0\,1)$. So general: $(-2\,0\,1) + a(-3\,1\,0)$.

Problem 2:

(a) Is $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{pmatrix}$ invertible? If so, find A^{-1}.

(b) Give the LDU decomposition of $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 6 & 3 & 5 \end{pmatrix}$.

(a) This is product of elementary matrices $A_3^{2\times 1} A_3^{3\times 2}$, so inverse is

$$(A_3^{3\times 2})^{-1}(A_3^{2\times 1})^{-1} = A_3^{-3\times 2} A_3^{-2\times 1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -3 & 1 \end{pmatrix}.$$

(b) $\overset{A_3^{-3\times 1}}{\rightarrow} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{pmatrix}$, *so* $D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ *and* $U = \begin{pmatrix} 1 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 1 \end{pmatrix}$.

And $L = (A_3^{-3\times 1})^{-1} = A_3^{3\times 1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$

Problem 3: Let V be the space of real-valued functions of x.
Show the solution set S of $f'(x) = xf(x)$ is a subspace of V.

If $f, g \in S$ then $f' = xf, g' = xg$.

So $(f+g)' = f' + g' = xf + xg = x(f+g)$; and also $f + g \in S$.

Similarly if $f \in S$ and c is scalar, $(cf)' = c(f') = c(xf) = x(cf)$ so that also $cf \in S$.

Problem 4:

(a) For $A = \begin{pmatrix} 1 & 1 & 0 & 2 \\ 1 & 3 & 1 & 2 \\ 3 & 1 & -1 & 6 \end{pmatrix}$ find a basis for the row space and the column space.
Is $Ax = b$ solvable for all b?

(b) Let V be the space of polynomials in x of degree at most 3.
Give a basis for the subspace W of such polynomials with $f(2) = 0$.

(a) (rows) $\overset{A_2^{-1\times 1}, A_3^{-3\times 1}}{\to} \begin{pmatrix} 1 & 1 & 0 & 2 \\ 0 & 2 & 1 & 0 \\ 0 & -2 & -1 & 0 \end{pmatrix}$

Then $A_3^{1\times 2}$ *converts third row to 0, so* $(1\,1\,0\,2)$ *and* $(0\,2\,1\,0)$ *give basis.*

(columns) Work on $A^T = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 1 \\ 0 & 1 & -1 \\ 2 & 2 & 6 \end{pmatrix} \overset{A_2^{-1\times 1}, A_4^{-2\times 1}}{\to} \begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{pmatrix}$.

This time $A_3^{-\frac{1}{2}\times 2}$ *kills third row; so basis is* $(1\,1\,3)^T$ *and* $(0\,2\,-2)^T$.
And $Ax = b$ *solvable only for* b *in this column space—here, NOT all* b.
(b) Easiest to note that V *is 4-dimensional with basis* $\{1, x, x^2, x^3\}$,
and W *is 3-dimensional subspace with* $x - 2$ *dividing* $f(x)$;
so a nice basis is $\{x - 2, x(x-2), x^2(x-2)\}$, *that is,* $\{x - 2, x^2 - 2x, x^3 - 2x^2\}$